Mechanized verification of type systems using Iris

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The old problem of proving "type safety":

"Well-typed programs cannot go wrong"

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Goal of this talk:

- Introduce the "logical approach in Iris" as an alternative to the standard progress/preservation approach to type safety
- Show that this approach makes it possible to type "unsafe" code
- Show that this approach is well-suited for mechanization of challenging type systems in the Coq proof assistant

Recap: Progress and preservation [Wright and Felleisen, simplified by Harper]

Safety is defined in terms of a small-step operational semantics:

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2. **Preservation:** If $\vdash e : A$ and $e \rightarrow e'$ then $\vdash e' : A$

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Proof of type safety: If $\vdash e : A$ then $\mathsf{safe}(e)$
Obtain $\vdash e' : A$ by induction on length of $e \rightarrow^* e'$ and preservation, conclude by progress

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  Disjointness conditions show up everywhere
  (And Coq does not accept "left as an exercise for the reader")

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- Even more tricky once you consider a substructural type system
  Disjointness conditions show up everywhere
  (And Coq does not accept "left as an exercise for the reader")
- Unsuitable to reason about "unsafe" code
  `unsafe` in Rust, `Obj.magic` in OCaml, `unsafePerformIO` in Haskell

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Key challenge: Define $\vDash e : A$ so that:

- It is rich enough to support challenging PL features
- It allows for a concise proof of the fundamental theorem

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- More abstract versions developed by Appel *et al.* (2007) and Dreyer *et al.* (2011)
- Iris provides a modern **logical approach** in which concurrent separation logic hides reasoning about state *and* which is well-suited for mechanized proofs

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And then change some conjunctions into separation conjunctions to scale to a substructural type system with channels implemented as an “unsafe” library

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application is not a value, we need to talk about its result

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Weakest precondition:

$$\mathsf{wp}\ \_\ \{\_\} : \mathsf{Expr} \to (\mathsf{Val} \to \mathsf{Prop}) \to \mathsf{Prop}$$

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closing substitution, I will ignore those most of the time

Semantic typing judgment:

$$\Gamma \vDash e : A \triangleq \forall \gamma.\, [\![\Gamma]\!]\, \gamma \Rightarrow \mathsf{wp}\ \gamma(e)\,\{[\![A]\!]\}$$

Proofs of key properties

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recall $[\![A \to B]\!] \triangleq \lambda v.\ \forall w.\ [\![A]\!]\ w \Rightarrow \mathsf{wp}\ (v\ w)\ \{[\![B]\!]\}$

An "unsafe" fixpoint combinator

Consider a strict version of Curry's fixpoint operator:

$$\textbf{fix} \triangleq \lambda f.\,(\lambda x.\, f\ (\lambda v.\, x\ x\ v))\ (\lambda x.\, f\ (\lambda v.\, x\ x\ v))$$

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✓ Yes. We can prove that **fix** is semantically safe

Towards "logical typing"

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separation logic

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Important observations:

- Each type is given a semantic interpretation following the Curry-Howard correspondence
- Instead of Coq's Prop we can use **a logic with more fancy connectives** to interpret more challenging types

concurrent separation logic

Towards "logical typing"

Recall the semantic interpretation of types ("logical relation"):

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higher-order concurrent separation logic

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Iris

Separation logic [O'Hearn, Reynolds, Yang; CSL'01]

Propositions P, Q denote ownership of resources

Separating conjunction $P * Q$:
The resources consists of separate parts satisfying P and Q

Basic example:

$$\{\ell_1 \mapsto v_1 * \ell_2 \mapsto v_2\} \mathit{swap}\ \ell_1\ \ell_2 \{\ell_1 \mapsto v_2 * \ell_2 \mapsto v_1\}$$

Separation logic [O'Hearn, Reynolds, Yang; CSL'01]

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the $*$ ensures that ℓ_1 and ℓ_2 are different memory locations

Separation logic [O'Hearn, Reynolds, Yang; CSL'01]

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Slightly less basic example:

$$\text{isList}\ \ell\ \vec{v} \triangleq \begin{cases} \ell \mapsto \text{nil} & \text{if } \vec{v} = [\,] \\ \exists \ell'.\, \ell \mapsto \text{cons}\ v_1\ \ell' * \text{isList}\ \ell'\ \vec{v}_2 & \text{if } \vec{v} = v_1 :: \vec{v}_2 \end{cases}$$

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$$\{\text{isList } \ell_1 \ \vec{v}_1 * \text{isList } \ell_2 \ \vec{v}_2\}\, \mathit{append}\ \ell_1 \ \ell_2 \{\text{isList } \ell_1 \ (\vec{v}_1 \mathbin{++} \vec{v}_2)\}$$

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the $*$ ensures that all nodes of ℓ_1 and ℓ_2 are disjoint

The simple model of separation logic

The semantic domains:

$$\ell \in \mathsf{Loc} \triangleq \mathbb{N}$$

$$\sigma \in \mathsf{Heap} \triangleq \mathsf{Loc} \xrightarrow{\text{fin}} \mathsf{Val}$$

$$P, Q \in \mathsf{heapProp} \triangleq \mathsf{Heap} \to \mathsf{Prop}$$

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The connectives of separation logic:

$$\ell \mapsto v \triangleq \lambda \sigma.\, \sigma(\ell) = v$$

$$P \wedge Q \triangleq \lambda \sigma.\, P\sigma \wedge Q\sigma$$

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disjointness of heaps, hidden by $*$

Semantic typing for a substructural type system

Semantic interpretation of types ("logical relation"):

$$[\![\_]\!] : \mathsf{Type} \to \mathsf{SemType} \quad \text{where} \quad \mathsf{SemType} \triangleq \mathsf{Val} \to \mathsf{heapProp}$$

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Weakest precondition of separation logic:

$$\mathsf{wp}\ \_\ \{\_\} : \mathsf{Expr} \to (\mathsf{Val} \to \mathsf{heapProp}) \to \mathsf{heapProp}$$

$$\mathsf{wp}\ e\,\{\Phi\} \approx \lambda\sigma.\, \mathsf{safe}(\sigma, e) \wedge (\forall v, \sigma'.\, (\sigma, e) \to^* (v, \sigma') \Rightarrow \Phi\ v\ \sigma')$$

(Modulo "frame baking")

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Iris **invariant** $\boxed{P}$ $\approx$ knowledge that P holds at all times (invariantly)

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This scales—pick the right Iris features to interpret your favorite types

Typing "unsafe" code: One-shot channels

We can **implement** one-shot channels instead of adding them as primitives to our language (akin to using `unsafe` in Rust):

$$
\begin{aligned}
\texttt{new}\,() &\triangleq \texttt{let}\ c = \texttt{ref}(\texttt{None})\ \texttt{in}\ (c, c) \\
\texttt{send}\, c\, v &\triangleq c := \texttt{Some}\ v \\
\texttt{recv}\, c &\triangleq \texttt{match}\ !c\ \texttt{with} \\
&\qquad\ \ \texttt{None} \quad\ \Rightarrow \texttt{recv}\, c \\
&\qquad \mid \texttt{Some}\ v \Rightarrow \texttt{free}(c);\ v \\
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One-shot channels + recursive types allow one to embed the whole of higher-order binary session types [Jacobs, ECOOP'22]

$$\begin{aligned}\texttt{recv}\ c = {} & \texttt{match}\ !c\ \texttt{with} \\ & \quad \texttt{None} \Rightarrow \texttt{recv}\ c \\ & \mid \texttt{Some}\ v \Rightarrow \texttt{free}(c);\ v \\ & \texttt{end}\end{aligned}$$

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What would be good typed API for one-shot channels?

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What would be good typed API for one-shot channels?

$$\vDash \texttt{new} : () \multimap\; !A \times ?A \qquad\qquad \vDash \texttt{send} : !A \multimap A \multimap () \qquad\qquad \vDash \texttt{recv} : ?A \multimap A$$

Typing "unsafe" code: Recipe

1. Provide a separation logic API for the unsafe operations
   Used to give a logical interpretation $[\![\_]\!]$ of the typed API
2. Prove Hoare style specifications for the unsafe operations
   Used to prove the semantic typing rules

Separation logic API for one-shot channels

Recall the desired typing rules:

$$\vDash \texttt{new} : () \multimap\ !A \times ?A$$

$$\vDash \texttt{send} : !A \multimap A \multimap ()$$

$$\vDash \texttt{recv} : ?A \multimap A$$

The separation logic API:

$$\{\mathsf{True}\}\ \texttt{new}\ ()\ \{(c_1, c_2).\ \mathsf{IsChan}(c_1, \mathsf{Send}, \Phi) * \mathsf{IsChan}(c_2, \mathsf{Recv}, \Phi)\}$$

$$\{\mathsf{IsChan}(c, \mathsf{Send}, \Phi) * \Phi\ v\}\ \texttt{send}\ c\ v\ \{\mathsf{True}\}$$

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$\Phi : \mathsf{Val} \to \mathsf{iProp}$, so we can transfer resources

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The semantic typing rules for channels follow immediately from the Hoare rules

Verification of one-shot channel separation logic API

One-shot channel ownership defined using standard Iris methodology

$$\mathsf{IsChan}(c, \mathit{tag}, \Phi) \triangleq \ldots$$

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One-shot channel ownership defined using standard Iris methodology:

1. Model behavior as a state transition system (STS)

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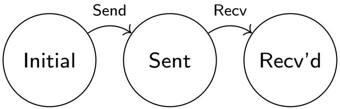

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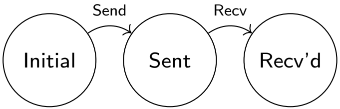

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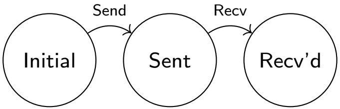

$$\text{chan\_inv} \triangleq (\underbrace{\qquad\qquad}_{\text{(1) initial state}}) \vee (\underbrace{\qquad\qquad\qquad\qquad\qquad}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\qquad\qquad\qquad}_{\text{(3) final state}})$$

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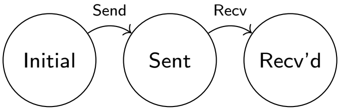
3. Determine resource ownership of each state



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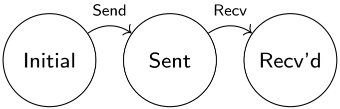

$$\text{chan\_inv} \quad c \quad \triangleq \underbrace{(\ c \mapsto \texttt{None}\ )}_{\text{(1) initial state}} \vee \underbrace{(\exists v.\, c \mapsto \texttt{Some}\, v \qquad)}_{\text{(2) message sent, but not yet received}} \vee \underbrace{(\qquad\qquad)}_{\text{(3) final state}}$$

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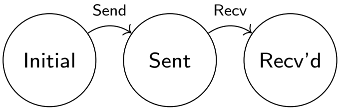

$$\text{chan\_inv} \qquad c\ \Phi \triangleq (\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \texttt{Some}\, v * \Phi\ v}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\qquad\qquad}_{\text{(3) final state}})$$

$$\text{IsChan}(c, \mathit{tag}, \Phi) \triangleq \ldots$$

Verification of one-shot channel separation logic API

One-shot channel ownership defined using standard Iris methodology:

1. Model behavior as a state transition system (STS)
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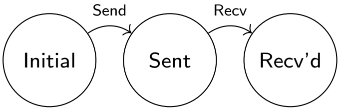

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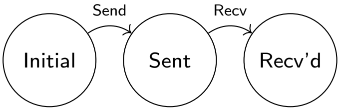

$$\text{chan\_inv}\ \gamma_s \quad c\ \Phi \triangleq (\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}) \lor (\underbrace{\exists v.\, c \mapsto \texttt{Some}\, v * \Phi\ v * \text{tok}\,\gamma_s}_{\text{(2) message sent, but not yet received}}) \lor (\underbrace{\qquad\qquad}_{\text{(3) final state}})$$

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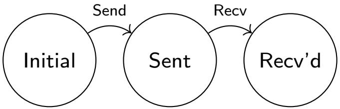

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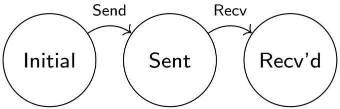

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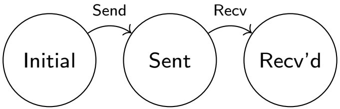

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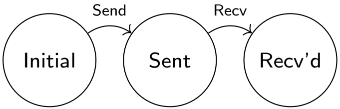

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Summary: Recipe for verifying a type system in Iris

1. Define the syntax and operational semantics for your language
2. Build a program logic using Iris, *i.e.*, define WP, $\mapsto$, *etc.*
3. Verify separation logic APIs for your “unsafe” libraries
4. Define a logical relation and semantic typing judgment
5. Prove semantic typing rules/fundamental theorem
6. Profit

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4. Define a logical relation and semantic typing judgment
   Interpret type formers using suitable logical connectives through Curry-Howard
5. Prove semantic typing rules/fundamental theorem
   Most of the heavy lifting is done by the Hoare/WP rules in Iris
6. Profit

The logical approach in Iris scales

Perennial DimSum Cerise RustBelt ReLoC

Melocoton VMSL RefinedC Aneris

Diaframe Iris-Wasm

Simuliris Compass

Iris-Tini RustHornBelt

Cosmo CQS SeLoC

iGPS Hazel gDOT GoJournal

OCPL Islaris Actris Iron iRC11

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How to do mechanized proofs in separation logic?

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Suppose we want to prove $P * (\exists a.\, \Phi a) * Q \ \vdash\ Q * (\exists a.\, P * \Phi a)$

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1. **Unfold definitions of the model**: $\forall \sigma.\, (\exists \sigma_1\, \sigma_2.\, \sigma = \sigma_1 \uplus \sigma_2 \wedge P \sigma_1 \wedge \ldots) \rightarrow \ldots$
   - Defeats the purpose of separation logic to hide reasoning about disjointness
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3. **Use Iris Proof Mode**
   - Topic of today's talk

Iris Proof Mode (IPM) [Krebbers *et al.*; POPL'17, ICFP'18]

Embedding of a proof assistant for separation logic in Coq

- Extend Coq with named proof contexts for separation logic
- Tactics for introduction and elimination of all connectives of separation logic ...
- ...that can be used in Coq's mechanisms for automation/tactic programming
- Implemented without modifying Coq (using reflection, type classes and Ltac)

Why use Coq and not build a standalone proof assistant for separation logic?

Prove soundness of embedded proof assistant

Reuse infrastructure of host proof assistant

Users do not need to learn new tool

Iris Proof Mode demo

```
Lemma test {A} (P Q : iProp) (Φ : A → iProp) :
  P ∗ (∃ a, Φ a) ∗ Q ⊢ Q ∗ ∃ a, P ∗ Φ a.
Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
  iSplitL "H3".
  - iAssumption.
  - iExists x.
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Qed.
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Lemma in separation logic

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```
1 subgoal
A : Type
P, Q : iProp
Φ : A → iProp
______________________________________(1/1)
P ∗ (∃ a : A, Φ a) ∗ Q
⊢ Q ∗ (∃ a : A, P ∗ Φ a)
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______________________∗
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∗ means: resources should be split

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The hypotheses for the left conjunct

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```
2 subgoals
A : Type
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Φ : A → iProp
x : A
______________________________________(1/2)
"H3" : Q
--------------------------------------∗
Q

______________________________________(2/2)
"H1" : P
"H2" : Φ x
--------------------------------------∗
∃ a : A, P ∗ Φ a
```

Iris Proof Mode demo

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We can also solve this goal automatically

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We can also solve this goal automatically

```
No more subgoals.
```

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Proof.
  iIntros "[$ [? $]] //".
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Or use intro patterns

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- **Integration with tactics for proving programs**
  Symbolic execution tactics for weakest preconditions
- **Tactic programming**
  One can combine/program with IPM tactics using Coq's Ltac like ordinary Coq tactics

Changes since the POPL'17 paper on Iris Proof Mode

- Generalized to any Bunched Implications (BI) logic (Krebbers *et al.*, ICFP'18)
- Many usability improvements:
  Smarter tactics, better error messages, improved robustness and performance
- Proof automation: **RefinedC** (Sammler *et al.* PLDI'21), **Diaframe** (Mulder *et al.* PLDI'22, PLDI'23, OOPSLA'23), BedRock Systems (proprietary)

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Most importantly: Iris (Proof Mode) got users:

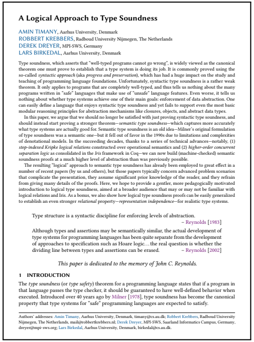
- Coq became essential to teach Iris / concurrent separation logic
- 13 PhD theses
- 98 publications
- 3 editions of the Iris workshop
- Used by researchers at companies: BedRock Systems, Meta, Jetbrains

Future work: Going beyond safety

- Applying the logical approach to deadlock freedom, resource leak freedom, liveness, non-interference remains challenging
- Different models of concurrent separation logic/Iris need to be explored: linear (instead of affine), transfinite, *etc.*
- We have initial versions for specific languages
- But we do not have the right Iris-style abstractions to build these logics modularly
- Nor to easily combine different PL features in one type safety proof

Read more?

Our overview:


A Logical Approach to Type Soundness

AMIN TIMANY, Aarhus University, Denmark
ROBBERT KREBBERS, Radboud University Nijmegen, The Netherlands
DEREK DREYER, MPI-SWS, Germany
LARS BIRKEDAL, Aarhus University, Denmark

Type soundness, which asserts that "well-typed programs cannot go wrong", is widely viewed as the canonical theorem one must prove to establish that a type system is doing its job. It is commonly proved using the so-called *syntactic approach* (aka *progress and preservation*), which has had a huge impact on the study and teaching of programming language foundations. Unfortunately, syntactic type soundness is a rather weak theorem. It only applies to programs that are completely well-typed, and thus tells us nothing about the many programs written in "safe" languages that make use of "unsafe" language features. Even worse, it tells us nothing about whether type systems achieve one of their main goals: enforcement of data abstraction. One can easily define a language that enjoys syntactic type soundness and yet fails to support even the most basic modular reasoning principles for abstraction mechanisms like closures, objects, and abstract data types.

In this paper, we argue that we should no longer be satisfied with just proving syntactic type soundness, and should instead start proving a stronger theorem—*semantic type soundness*—which captures more accurately what type systems are actually good for. Semantic type soundness is an old idea—Milner's original formulation of type soundness was a semantic one—but it fell out of favor in the 1990s due to limitations and complexities of denotational models. In the succeeding decades, thanks to a series of technical advances—notably, (1) *step-indexed Kripke logical relations* constructed over operational semantics and (2) *higher-order concurrent separation logic* as consolidated in the *Iris* framework in Coq—we can now build (machine-checked) semantic soundness proofs at a much higher level of abstraction than was previously possible.

The resulting "logical" approach to semantic type soundness has already been employed to great effect in a number of recent papers (by us and others), but those papers typically concern advanced problem scenarios that complicate the presentation, they assume significant prior knowledge of the reader, and they refrain from giving many details of the proofs. Here, we hope to provide a gentler, more pedagogically motivated introduction to logical type soundness, aimed at a broader audience that may or may not be familiar with logical relations and Iris. As a bonus, we also show how logical type soundness proofs can be easily generalized to establish an even stronger *relational* property—*representation independence*—for realistic type systems.

> Type structure is a syntactic discipline for enforcing levels of abstraction.
> – Reynolds [1983]

> Although types and assertions may be semantically similar, the actual development of type systems for programming languages has been quite separate from the development of approaches to specification such as Hoare logic… the real question is whether the dividing line between types and assertions can be erased. – Reynolds [2002]

This paper is dedicated to the memory of John C. Reynolds.

1 INTRODUCTION

The *type soundness* (or *type safety*) theorem for a programming language states that if a program in that language passes the type checker, it should be guaranteed to have well-defined behavior when executed. Introduced over 40 years ago by Milner [1978], type soundness has become the canonical property that type systems for "safe" programming languages are expected to satisfy.

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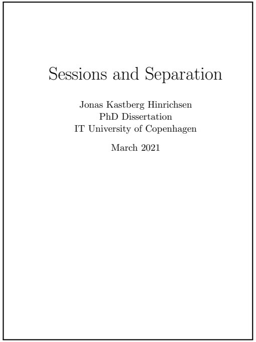
Session types:


Sessions and Separation

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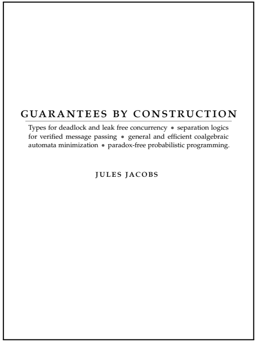
PhD Dissertation
IT University of Copenhagen

March 2021

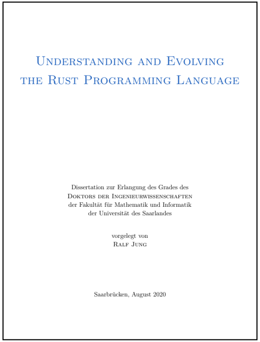
Deadlock freedom:


GUARANTEES BY CONSTRUCTION

Types for deadlock and leak free concurrency • separation logics for verified message passing • general and efficient coalgebraic automata minimization • paradox-free probabilistic programming.

JULES JACOBS

Rust:


UNDERSTANDING AND EVOLVING THE RUST PROGRAMMING LANGUAGE

Dissertation zur Erlangung des Grades des
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