Separation Logic for Non-local Control Flow and Block Scope Variables

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Joint work with Freek Wiedijk

Radboud University Nijmegen

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What is this program supposed to do?

```
int *p = NULL;
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} else {
  int j = 10;
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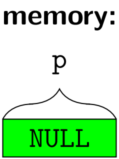

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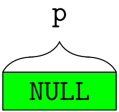

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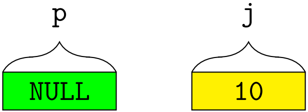

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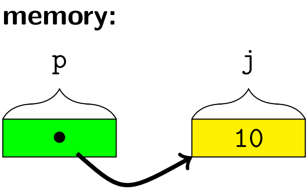

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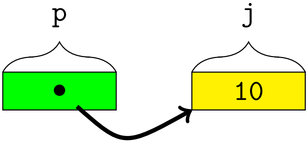

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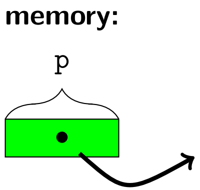

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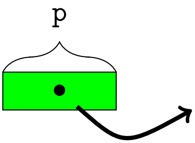

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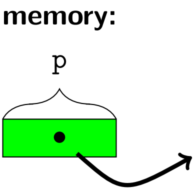

It exhibits undefined behavior, thus it may do *anything*

Undefined behavior in C

- *Undefined behavior* is shown by “wrong” C programs
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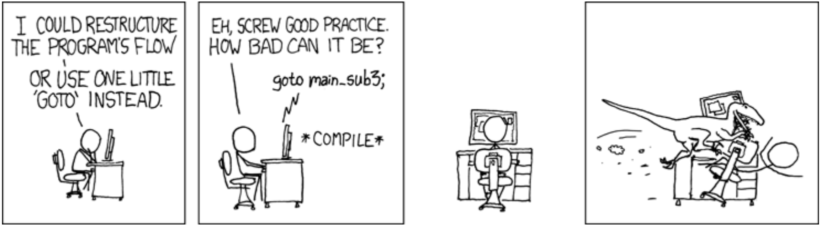
This talk: undefined behavior due to dangling pointers by non-local control flow and block scopes

Goto considered harmful?



http://xkcd.com/292/

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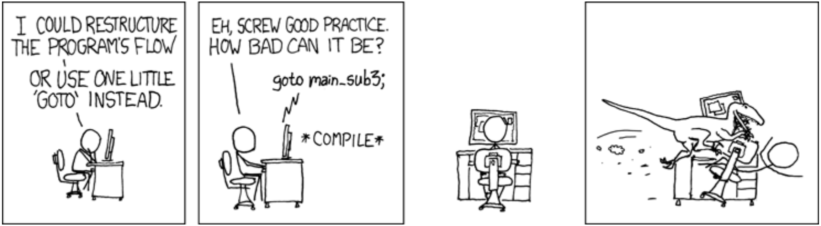

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Not necessarily:

$$\vdash \{P\} \ldots \ \texttt{goto main\_sub3;} \ \ldots \{Q\}$$

Contribution

A concise small step operational, and axiomatic, semantics for goto, supporting:

- local variables (and pointers to those),
- mutual recursion,
- separation logic,
- soundness proof fully checked by Coq

Approach

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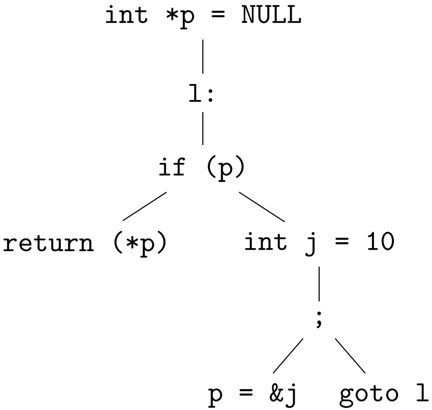
- Traversal through the AST in the following directions:
  - $\searrow$ downwards to the next statement
  - $\nearrow$ upwards to the next statement
  - $\curvearrowright$`l` to a label `l:` after a `goto l`
  - $\upuparrows$ to the top of the statement after a `return`

Example



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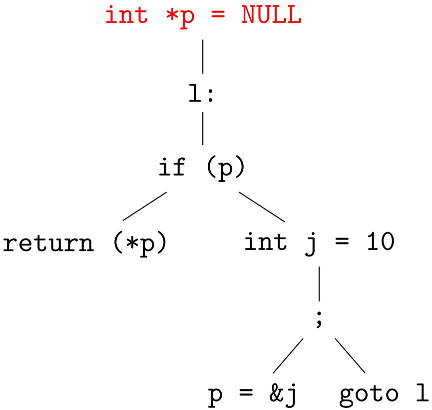
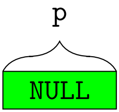


direction:

↘

memory:



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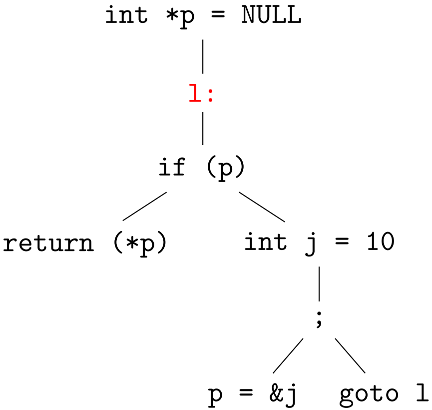

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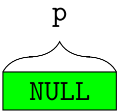

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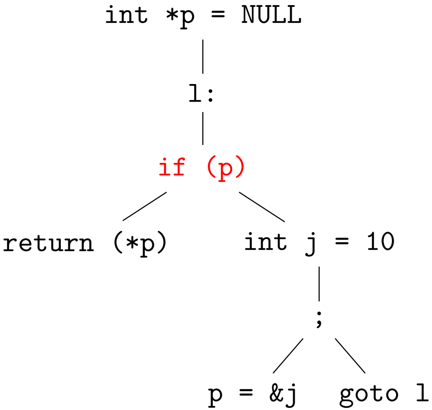

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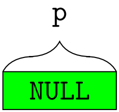

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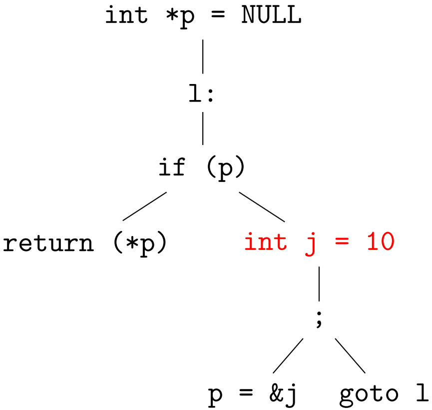

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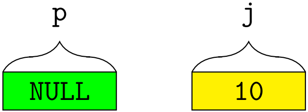

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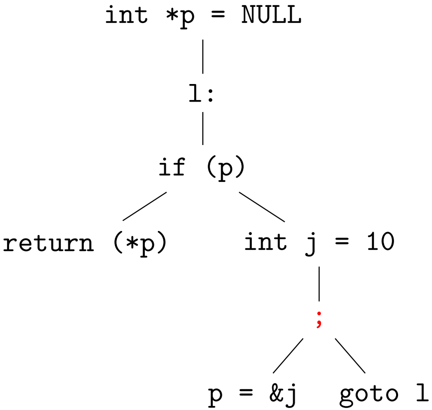

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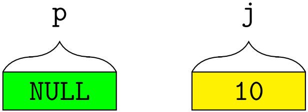

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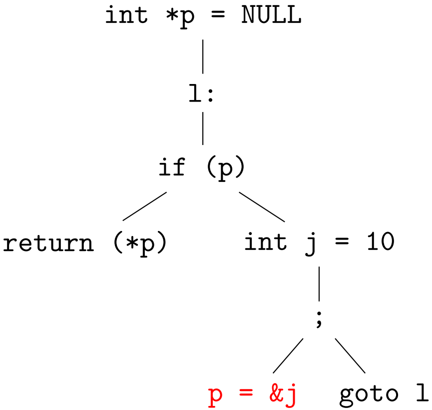

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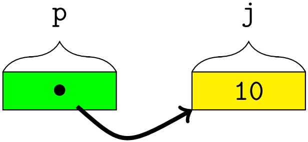

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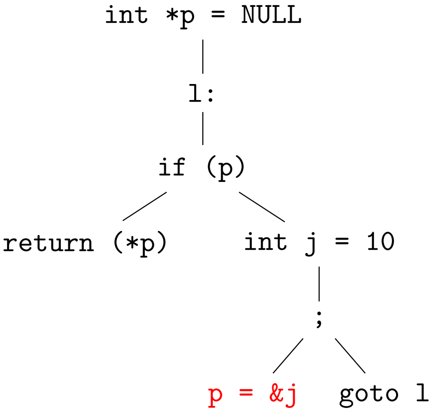

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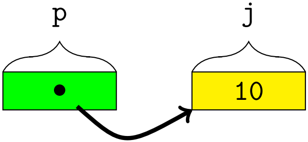

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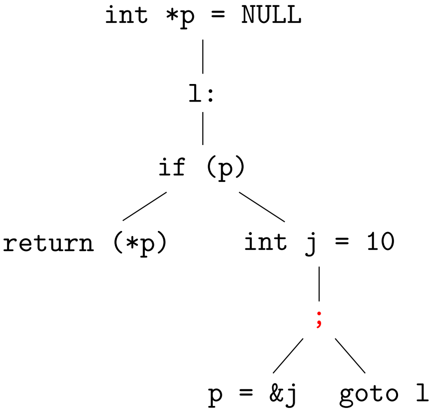

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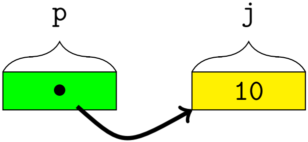

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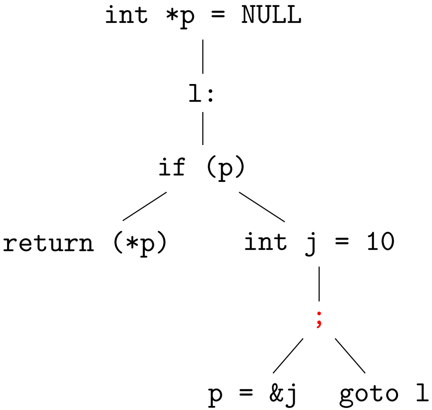

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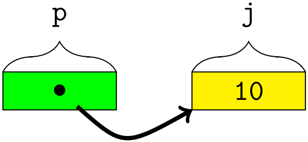

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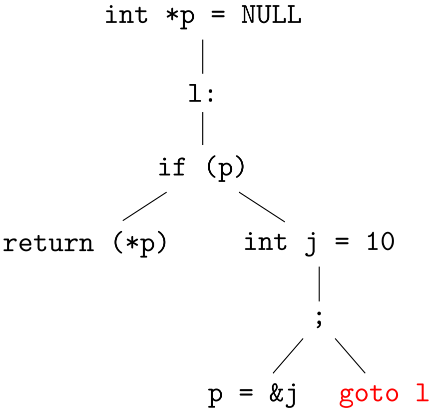

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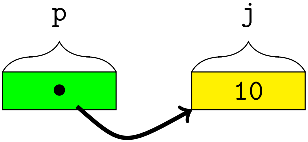

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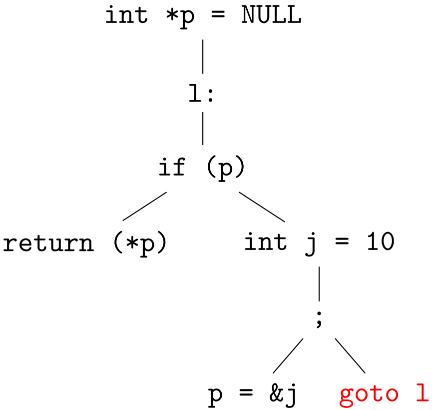

direction:

$\curvearrowright$ `l`

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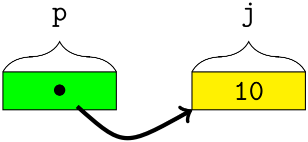

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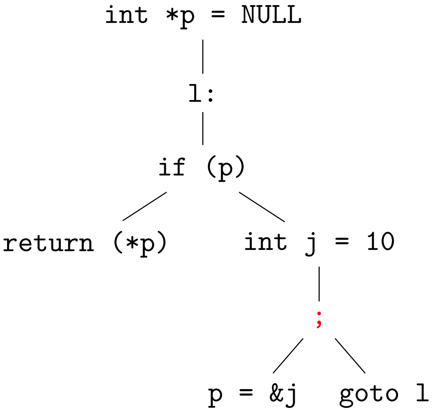

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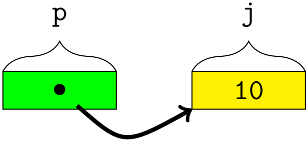

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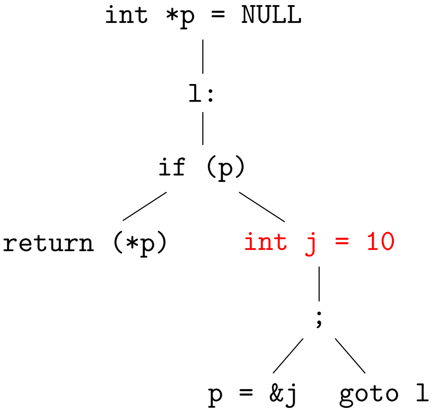

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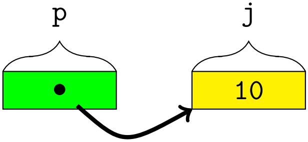

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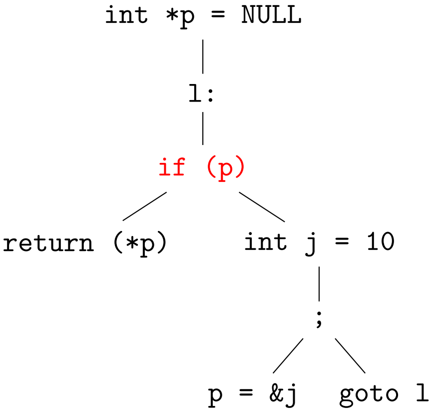

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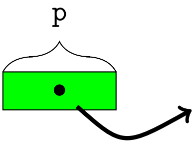

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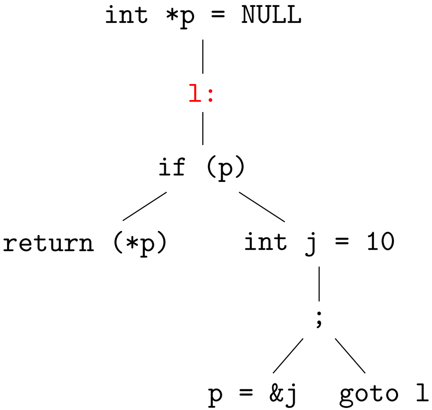

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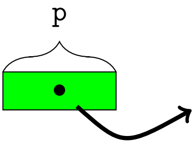

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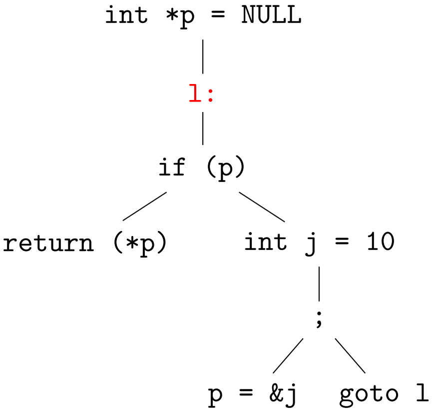

direction:

$\searrow$

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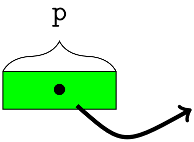

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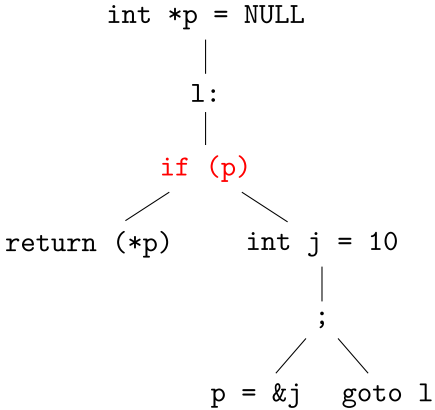

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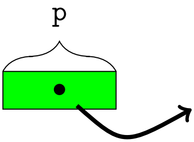

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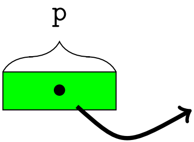
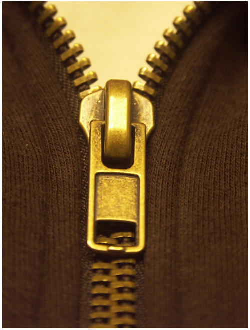


How to model the current *location* in the program

Huet's zipper



Purely functional way to store a pointer into a data structure

Statement contexts

- Statements:

$$s ::= \texttt{block}\ s \mid e_l := e_r \mid f(\vec{e}) \mid \texttt{skip} \mid \texttt{goto}\ l$$
$$\mid l : s \mid s_1 \,;\, s_2 \mid \texttt{if}\ (e)\ s_1\ s_2 \mid \texttt{return}$$

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$$E_{\mathcal{S}} ::= \square \,;\, s_2 \mid s_1 \,;\, \square \mid \texttt{if}\; (e)\; \square\; s_2 \mid \texttt{if}\; (e)\; s_1\; \square \mid l : \square$$

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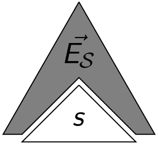
- A pair $(\vec{E_{\mathcal{S}}}, s)$ forms a zipper for statements, where
  - $\vec{E_{\mathcal{S}}}$ is a statement turned inside-out
  - s is the focused substatement



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- Extend the zipper dynamically on function calls
- Program contexts k are lists of singular program contexts:

$$E ::= E_{\mathcal{S}} \mid \texttt{block}_b\,\Box \mid \ \ldots$$

where $\texttt{block}_b\,\Box$ associates a block scope variable with its corresponding memory index b

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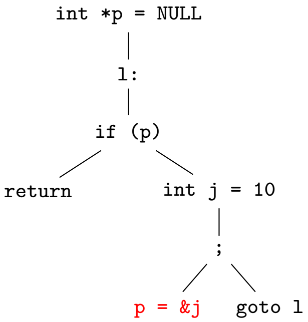
- (d, s) execution of a statement s in direction d
- $\overline{\text{call}}\, f\ \vec{v}$ calling a function $f(\vec{v})$
- $\overline{\text{return}}$ returning from a function

Example



The corresponding state is $\mathbf{S}(k, \phi, m)$, where:

- $k = [$
  $\square \,;\, \texttt{goto}\ l,$
  $x_0 := \texttt{int}\ 10 \,;\, \square,$
  $\texttt{block}_{b_j}\ \square,$
  $\texttt{if}\ (\texttt{load}\ x_0)\ \texttt{return}\ \square,$
  $l : \square,$
  $x_0 := \texttt{NULL} \,;\, \square,$
  $\texttt{block}_{b_p}\ \square$
  $]$
- $\phi = (\nearrow, x_1 := x_0)$
- $m = \{b_p \mapsto \texttt{ptr}\ b_j, b_j \mapsto 10\}$

The small step semantics

Lemma

The small step semantics behaves as traversing through a zipper. That is, if

$$\mathbf{S}(k, (d, s), m) \rightharpoonup_k^* \mathbf{S}(k, (d', s'), m')$$

then $s = s'$.

The small step semantics

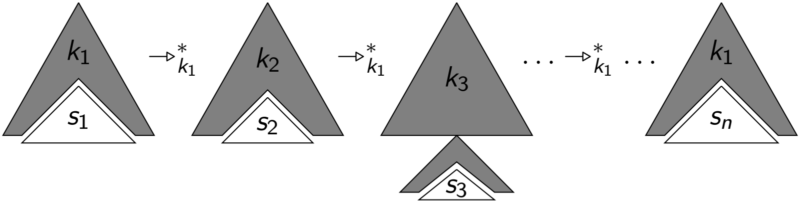
Lemma

The small step semantics behaves as traversing through a zipper. That is, if

$$\mathbf{S}(k,\,(d,\,s),\,m) \rightharpoonup^*_k \mathbf{S}(k,\,(d',\,s'),\,m')$$

then $s = s'$.

In a picture: if



then $s_1 = s_n$.

Hoare sextuples

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Remark: the assertions P, Q, J and R correspond to the directions $\searrow$, $\nearrow$, $\curvearrowright$ and $\upuparrows$ of traversal

Some Hoare rules

Composition:

$$\frac{\Delta;\ J;\ R \vdash \{P\}\, s_1\, \{P'\} \qquad \Delta;\ J;\ R \vdash \{P'\}\, s_2\, \{Q\}}{\Delta;\ J;\ R \vdash \{P\}\, s_1\, ; s_2\, \{Q\}}$$

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Goto:

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Return:

$$\frac{}{\Delta;\ J;\ R \vdash \{R\}\, \texttt{return}\, \{Q\}}$$

The frame rule

Used for local reasoning

$$\frac{\Delta;\ J;\ R \vdash \{P\}\, s\, \{Q\}}{\Delta;\ J * A;\ R * A \vdash \{P * A\}\, s\, \{Q * A\}}$$

The block scope variable rule

$$\frac{\Delta;\, J\uparrow * x_0 \mapsto \text{-};\; R\uparrow * x_0 \mapsto \text{-} \vdash \{P\uparrow * x_0 \mapsto \text{-}\}\, s\, \{Q\uparrow * x_0 \mapsto \text{-}\}}{\Delta;\, J;\, R \vdash \{P\}\, \texttt{block}\, s\, \{Q\}}$$

When entering a block:

- The De Bruijn indexes are lifted: $(\_)\uparrow$
- The memory is extended: $(\_) * x_0 \mapsto \text{-}$

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Important: using De Bruijn indexes avoids shadowing

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- Uses lots of automation
- 3500 lines of code

Proved in Coq

Future research

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Questions

Sources: `http://robbertkrebbers.nl/research/ch2o/`