MoSeL: A General, Extensible Modal Framework for Interactive Proofs in Separation Logic

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[1]MoSeL is joint work with Jacques-Henri Jourdan, Ralf Jung, Joseph Tassarotti, Jan-Oliver Kaiser, Amin Timany, Arthur Charguéraud, and Derek Dreyer

Separation logic [O'Hearn, Reynolds, and Yang, 2001]

Propositions P, Q denote ownership of resources

Separating conjunction $P * Q$:
The resources consists of separate parts satisfying P and Q

Basic example:

$$\{x \mapsto v_1 * y \mapsto v_2\} swap(x, y) \{x \mapsto v_2 * y \mapsto v_1\}$$

the $*$ ensures that x and y are different memory locations

Why is separation logic useful?

Separation logic is very useful:

- It provides a high level of modularity
- It scales to fancy PL features like concurrency

Just in Coq, there is an ever growing collection of separation logics:

- Bedrock
- CFML
- Charge!
- CHL
- FCSL
- Iris
- VST
- ...

The challenge

When developing a new separation logic in a proof assistant, one has to:

1. Prove soundness
2. Develop tactics to carry out proofs

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When developing a new separation logic in a proof assistant, one has to:

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These steps are tedious, can we simplify them?

In prior work, we proposed solutions for both problems:

1. Proving soundness: **Iris** [POPL'15, ICFP'16, ESOP'17, JFP'18]
2. Tactics: **Iris Proof Mode** [POPL'17]

Iris [POPL'15, ICFP'16, ESOP'17, JFP'18]

A general, language-independent, framework for modeling your own domain specific higher-order separation logics

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A general, language-independent, framework for modeling your own domain specific higher-order separation logics

- **General:** unifies the reasoning principles in many other logics

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- **General:** unifies the reasoning principles in many other logics
- **Language-independent:** parameterized by the language

Iris [POPL'15, ICFP'16, ESOP'17, JFP'18]

A general, language-independent, framework for modeling your own domain specific higher-order separation logics

- **General:** unifies the reasoning principles in many other logics
- **Language-independent:** parameterized by the language
- **Modeling logics:** can be used to model domain specific logics
  - iGPS for weak memory [ECOOP'17]
  - RustBelt's lifetime logic [POPL'18]
  - ReLoC for program refinements [LICS'18]
  - Iron for resource management [manuscript]

Iris Proof Mode [Krebbers *et al.*,POPL'17]: Coq tactics for Iris

```
Lemma test {A} (P Q : iProp) (Ψ : A → iProp) :
  P ∗ (∃ a, Ψ a) ∗ Q −∗ Q ∗ ∃ a, P ∗ Ψ a.
Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
  iSplitL "H3".
  - iAssumption.
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    iFrame.
Qed.
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Lemma in the Iris logic

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```

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"H2" : Ψ x
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∗ means: resources should be split

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The hypotheses for the left conjunct

```
Qed.
```

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```

The hypotheses for the left conjunct

```
Qed.
```

```
2 subgoals
A : Type
P, Q : iProp
Ψ : A → iProp
x : A
______________________________________(1/2)
"H3" : Q
--------------------------------------∗
Q

______________________________________(2/2)
"H1" : P
"H2" : Ψ x
--------------------------------------∗
∃ a : A, P ∗ Ψ a
```

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Proof.
  iIntros "[H1 [H2 H3]]".
  by iFrame.
Qed.
```

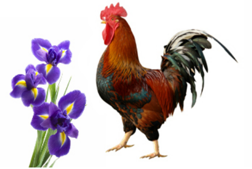
We can also solve this lemma automatically

```
No more subgoals.
```

The **good things** about Iris Proof Mode

It enabled mechanized proofs in many papers because:

- **Proofs have the look and feel of ordinary Coq proofs**
  For many Coq tactics `tac`, it has a variant `iTac`



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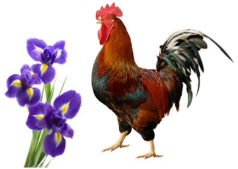
  Higher-order quantification, invariants, ghost state, later $\triangleright$ modality, ...



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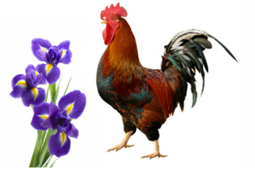
- **Integration with tactics for proving programs**
  Symbolic execution tactics for weakest preconditions



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- **Support for advanced features of separation logic**
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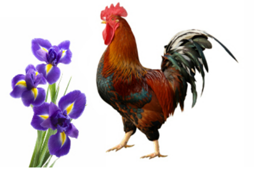
- **Integration with tactics for proving programs**
  Symbolic execution tactics for weakest preconditions
- **Usable in practice**
  Used for any project involving Iris today



The **bad thing** about Iris Proof Mode

The implementation is tied to Iris

~~Iris~~ Proof Mode

Making Iris Proof Mode independent of Iris

It sounds easy **[Krebbers et al., POPL'17]**:

> [. . .] we believe that our proof mode is very generic, and can be applied to a variety of different embedded logics [. . .]

Making Iris Proof Mode independent of Iris

It sounds easy [Krebbers et al., POPL'17]:

[...] we believe that our proof mode is very generic, and can be applied to a variety of different embedded logics [...]

But doing it generally will be be more challenging

Problem #1: Iris propositions are affine

In Iris you may "forget" about resources:

$$\{\ell_1 \mapsto v_1 * \ell_2 \mapsto v_2\}\, \ell_2 := !\, \ell_1\, \{\ell_2 \mapsto v_1\}$$

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Due to the **affinity axiom** $P * Q \vdash Q$, which is hard-wired into many tactics:

`iClear`

$$\frac{\Pi \Vdash Q}{\Pi, P \Vdash Q}$$

`iAssumption`

$$\Pi, P \Vdash P$$

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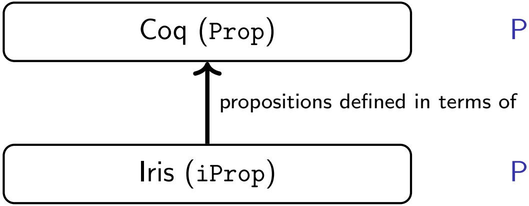
`iAssumption`

$$\Pi, P \Vdash P$$

Not having the affinity axiom is useful: precise accounting of resources (recall Aleš's talk)

Challenge: How to disentangle the affinity axiom from the Iris tactics?

Problem #2: No tactical support for derived logics



Proof using standard Coq tactics

Proof using Iris tactics

Problem #2: No tactical support for derived logics

Coq (`Prop`) — Proof using standard Coq tactics

↑ propositions defined in terms of

Iris (`iProp`) — Proof using Iris tactics

↑ propositions defined in terms of, $\mathrm{iGpsProp} \triangleq \mathrm{View} \xrightarrow{\text{mon}} \mathrm{iProp}$

Derived logic (*e.g.* `iGpsProp`)

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Challenge: How to reason in logics defined in terms of another

Contributions

MoSeL: A General, Extensible Modal Framework for Interactive Proofs in Separation Logic in Coq [Krebbers *et al.*,ICFP'18]

Contributions:

- MoSeL is parameterized by a general abstraction of separation logic
- MoSeL supports general and affine separation logics, and combinations thereof
- MoSeL supports reasoning in derived separation logics
- MoSeL can be fine-tuned for each logic using type classes

[2] Not in the ICFP'18 paper

Contributions

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MoSeL is usable in practice: we used it on 6 very different existing separation logics

CFML CHL Fairis iGPS Iris Iron[2]

[2] Not in the ICFP'18 paper

Implementation of tactics in IPM/MoSeL

Separation logic entailments in Coq

```
Lemma test {A} (P Q : iProp) (Ψ : A → iProp) :
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Notation for deeply embedded context

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--------------------------------∗
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envs_entails (Envs Enil
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   (String (Ascii false false
    false true false false true
    false)
   (String (Ascii true false
    false false true true false
    false)
   EmptyString)) P)
   ...
```

How to embed separation logic entailments into Coq?

Visible goal (with pretty printing):

$\vec{\mathtt{x}} : \vec{\phi}$ Variables and pure Coq hypotheses

———————————————————————————

Π Spatial separation logic hypotheses

———————————————————————————∗

R Separation logic goal

How to embed separation logic entailments into Coq?

Visible goal (with pretty printing):

$$\begin{array}{ll}
\vec{x} : \vec{\phi} & \text{Variables and pure Coq hypotheses} \\
\hline
\Pi & \text{Spatial separation logic hypotheses} \\
\hline
R & \text{Separation logic goal}
\end{array}$$

Actual Coq goal (without pretty printing):

$$\begin{array}{l}
\vec{x} : \vec{\phi} \\
\hline
\Pi \Vdash Q
\end{array}$$

Where:

$$\Pi \Vdash Q \triangleq \mathop{\Huge *} \Pi \vdash Q$$

Implementation of the `iSplitL`/`iSplitR` tactic

Tactics implemented by reflection as mere lemmas:

```
Lemma tac_sep_split Π Π₁ Π₂ lr js Q₁ Q₂ :
  envs_split lr js Π = Some (Π₁,Π₂) →
  (Π₁ ⊢ Q₁) → (Π₂ ⊢ Q₂) → Π ⊢ Q₁ ∗ Q₂.
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$$\frac{\Pi_1 \Vdash Q_1 \qquad \Pi_2 \Vdash Q_2}{\Pi_1, \Pi_2 \Vdash Q_1 * Q_2}$$

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Ltac wrappers around the reflective tactic:

```
Tactic Notation "iSplitL" constr(Hs) :=
  let Hs := words Hs in
  let Hs := eval vm_compute in (INamed <$> Hs) in
  eapply tac_sep_split with _ _ Left Hs _ _;
    [pm_reflexivity ||
      fail "iSplitL: hypotheses" Hs "not found"
    | (* goal 1 *)
    | (* goal 2 *) ].
```

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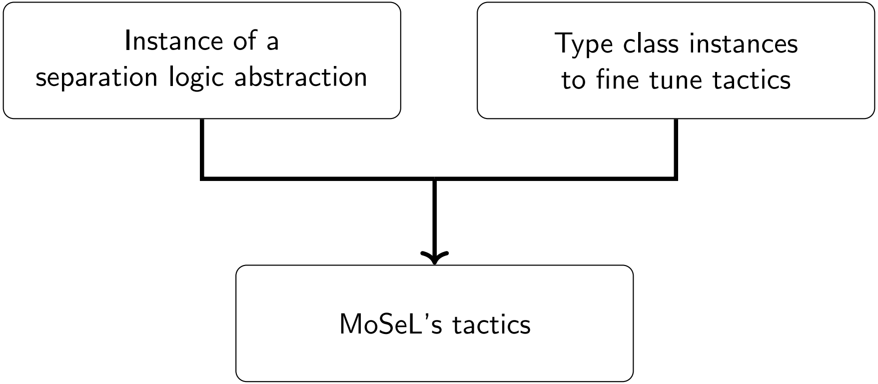
Report sensible error to the user

Making MoSeL separation logic independent



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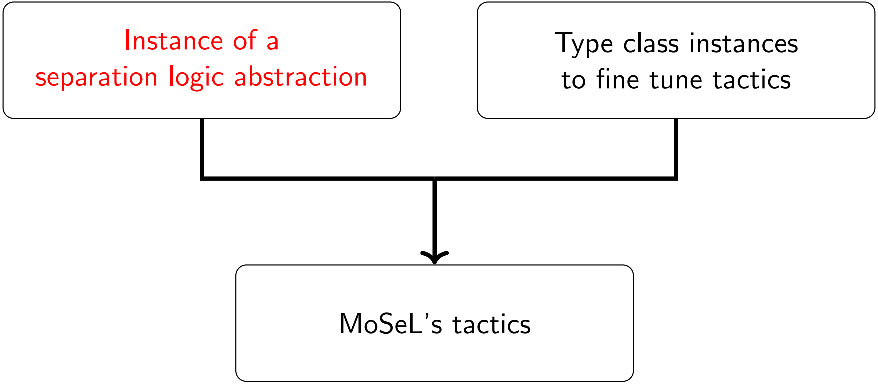

How to abstract over separation logics?

BI logics: An abstract interface for separation logics

A **Bunched Implications (BI) logic** [O'Hearn&Pym,99] is a preorder $(Prop, \vdash)$ with:

- Operations $\mathsf{True}, \mathsf{False}, \wedge, \vee, \Rightarrow, \forall, \exists$ satisfying the axioms of intuitionistic logic
- Operations $\mathsf{emp}, *, -\!\!*$ satisfying:

$$\begin{aligned} \mathsf{emp} * P &\dashv\vdash P \\ P * Q &\vdash Q * P \\ (P * Q) * R &\vdash P * (Q * R) \end{aligned} \qquad \frac{P_1 \vdash Q_1 \qquad P_2 \vdash Q_2}{P_1 * P_2 \vdash Q_1 * Q_2} \qquad \frac{P * Q \vdash R}{\overline{P \vdash Q -\!\!* R}}$$

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$$\begin{array}{rcl} \mathsf{emp} * P & \dashv\vdash & P \\ P * Q & \vdash & Q * P \\ (P * Q) * R & \vdash & P * (Q * R) \end{array} \qquad \frac{P_1 \vdash Q_1 \qquad P_2 \vdash Q_2}{P_1 * P_2 \vdash Q_1 * Q_2} \qquad \frac{P * Q \vdash R}{\overline{\overline{P \vdash Q \mathrel{-\!\!*} R}}}$$

```
Structure bi := Bi {
  bi_car     :> Type;
  bi_entails : bi_car → bi_car → Prop;
  bi_forall  : ∀ A, (A → bi_car) → bi_car;
  bi_sep     : bi_car → bi_car → bi_car;
  (* other separation logic operators and axioms *)
}.
```

Proofs in MoSeL

Proofs in a specific logic:

```
Lemma test {A} (P Q : iGpsProp) (Ψ : A → iGpsProp) :
  P ∗ (∃ a, Ψ a) ∗ Q −∗ Q ∗ ∃ a, P ∗ Ψ a.
Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
  iSplitL "H3".
  - iAssumption.
  - iExists x.
    iFrame.
Qed.
```

Proofs for all logics:

```
Lemma test {PROP : bi} {A} (P Q : PROP) (Ψ : A → PROP) :
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Proof.
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Proofs in MoSeL

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Qed.
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Lemma for another logic than Iris

Proofs for all logics:

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  – iAssumption.
  – iExists x.
    iFrame.
Qed.
```

Proofs in MoSeL

Proofs in a specific logic:

```
Lemma test {A} (P Q : iGpsProp) (Ψ : A → iGpsProp) :
  P ∗ (∃ a, Ψ a) ∗ Q −∗ Q ∗ ∃ a, P ∗ Ψ a.
Proof.
  iIntros "[H1 [H2 H3]]".
    iAssumption.
  – iExists x.
    iFrame.
Qed.
```

Lemma for another logic than Iris

Proofs for all logics:

```
Lemma test {PROP : bi} {A} (P Q : PROP) (Ψ : A → PROP) :
  P ∗ (∃ a, Ψ a) ∗ Q −∗ Q ∗ ∃ a, P ∗ Ψ a.
Proof.
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```

Lemma universally quantified in the BI logic

Addressing challenge #1:

Disentangling the affinity axiom

$$P * Q \vdash Q$$

A poor man's solution

Make two versions of the tactics

1. For affine logics (like Iris and iGPS)
2. For non-affine logics (like CFML and CHL)

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2. For non-affine logics (like CFML and CHL)

Problems:

- Duplicate work/maintenance
- Some logics mix affine and non-affine propositions, for example:

GC locations (affine)	**Non-GC locations** (not affine)
$\ell \mapsto_{\mathrm{gc}} v$	$\ell \mapsto v$

(Another example in [Tassarotti *et al.*, ESOP'17])

Key idea

- **Don't**: classify whether the whole logic is affine
- **Do:** classify whether individual propositions are affine

Classifying whether propositions are affine

Affine propositions:

$$\mathrm{affine}(P) \triangleq P \vdash \mathrm{emp}$$

(propositions that can be "thrown away")

The new tactics:

$$\texttt{iClear}\ \frac{\Pi \Vdash Q \qquad \mathrm{affine}(P)}{\Pi, P \Vdash Q}$$

$$\texttt{iAssumption}\ \frac{\mathrm{affine}(\Pi)}{\Pi, Q \Vdash Q}$$

Classifying whether propositions are affine in Coq

A new type class:

```
Class Affine {PROP : bi} (Q : PROP) := affine : Q ⊢ emp.
```

Instances:

- Tell MoSeL that specific connectives are affine:

  ```
  Instance mapsto_gc_affine l v : Affine (l ↦gc v).
  ```

- Capture that affine propositions are closed under most connectives:

  ```
  Instance sep_affine {PROP : bi} (P Q : bi) :
    Affine P → Affine Q → Affine (P ∗ Q).
  ```

MoSeL: A General, Extensible Modal Framework for Interactive Proofs in Separation Logic in Coq

What about modalities?

The affine modality

The **affine modality**:

$$
\begin{aligned}
\langle \mathit{affine} \rangle\, P \quad &\triangleq \quad P \wedge \mathsf{emp} \\
&\approx \quad \text{“}P\text{ holds using just affine resources”}
\end{aligned}
$$

This modality is useful for reasoning about affinity

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- Can be used to turn any proposition into an affine version, *e.g.* A wand that can be dropped $\langle \mathit{affine} \rangle\, (P \mathrel{-\!\!*} Q)$

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  A wand that can be dropped $\langle \mathit{affine} \rangle\, (P \mathrel{-\!\!*} Q)$
- Commutes with most operators, *e.g.*
  $\langle \mathit{affine} \rangle\, (P \vee Q) \dashv\vdash \langle \mathit{affine} \rangle\, P \vee \langle \mathit{affine} \rangle\, Q$

The affine modality

The **affine modality**:

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- Can be used to turn any proposition into an affine version, *e.g.* A wand that can be dropped $\langle \mathit{affine} \rangle \, (P \mathrel{-\!\!*} Q)$
- Commutes with most operators, *e.g.* $\langle \mathit{affine} \rangle \, (P \vee Q) \dashv\vdash \langle \mathit{affine} \rangle \, P \vee \langle \mathit{affine} \rangle \, Q$
- Gives rise to an alternative classification of affine propositions $\mathsf{affine}(P) \quad \text{iff} \quad P \vdash \langle \mathit{affine} \rangle \, P$

The idea of carving out classes of propositions and defining their corresponding modalities is widely applicable:

- Persistent propositions $\boxdot$
- Intuitionistic propositions $\square$
- Absorbing propositions $\langle \mathit{absorb} \rangle$
- Timeless propositions (in step-indexed logics) $\triangleright, \diamond$
- Objective propositions (in iGPS) $\langle \mathit{obj} \rangle, \langle \mathit{subj} \rangle$
- Normal propositions (in CFML) $\langle \mathit{normal} \rangle$
- ...

The paper shows how to modularly deal with such classes and use them in general tactics

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The paper shows how to modularly deal with such classes and use them in general tactics, we discuss two

Persistent and intuitionistic propositions

	Not droppable	**Droppable**
Not duplicable	Any	Affine
Duplicable	Persistent	Intuitionistic = (affine & persistent)

Persistent and intuitionistic propositions

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Duplicable	Persistent	Intuitionistic = (affine & persistent)

Intuitionistic propositions:

- Widely used in practice, *e.g.* for reasoning about persistent (and garbage collected) data structures
- MoSeL has special support for them

Intuitionistic propositions in MoSeL

```
Lemma test {PROP : bi} {A}
    (P Q : PROP) (Ψ : A → PROP) :
  P ∗ □ (∃ a, Ψ a) -∗ ∃ a, Ψ a ∗ (P ∗ Ψ a).
Proof.
  iIntros "[H1 #H2]".
  iDestruct "H2" as (x) "H2".
  iExists x.
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  - by iFrame.
Qed.
```

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Qed.
```

```
1 subgoal
PROP : bi
A : Type
P, Q : PROP
Ψ : A → PROP
________________________(1/1)
"H2" : ∃ a : A, Ψ a
________________________□
"H1" : P
________________________∗
∃ a : A, Ψ a ∗ (P ∗ Ψ a)
```

Moves hypothesis to **intuitionistic** context

Intuitionistic propositions in MoSeL

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Lemma test {PROP : bi} {A}
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```

Do not need to split intuitionistic context

```
1 subgoal
PROP : bi
A : Type
P, Q : PROP
Ψ : A → PROP
x : A
______________________(1/1)
"H2" : Ψ x
______________________□
"H1" : P
______________________∗
x ∗ (P ∗ Ψ x)
```

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Lemma test {PROP : bi} {A}
    (P Q : PROP) (Ψ : A → PROP) :
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  - by iFrame.
Qed.
```

```
2 subgoals
PROP : bi
A : Type
P, Q : PROP
Ψ : A → PROP
x : A
______________________________________(1/2)
"H2" : Ψ x
--------------------------------------□
Ψ x

______________________________________(2/2)
"H2" : Ψ x
--------------------------------------□
"H1" : P
--------------------------------------∗
P ∗ Ψ x
```

Formal treatment of the intuitionistic context

Visible Coq goal (with pretty printing):

$$\begin{array}{ll} \Gamma & \text{Intuitionistic separation logic hypotheses} \\ \hline\hline \Pi & \text{Spatial separation logic hypotheses} \\ \hline R & \text{Separation logic goal} \end{array}$$

Γ Intuitionistic separation logic hypotheses
——————————————————— $\square$
Π Spatial separation logic hypotheses
———————————————————$*$
R Separation logic goal

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Actual Coq goal (without pretty printing):

$$\Gamma ; \Pi \Vdash Q \quad \triangleq \quad \Box \left(\bigwedge \Gamma \right) * \left(\mathop{\Huge *} \Pi \right) \vdash Q$$

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Properties of the **intuitionistic context** Γ:

- Hypotheses can be duplicated (by tactics like `iSplitL`)

Formal treatment of the intuitionistic context

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- Hypotheses can be moved to the spatial context Π (by tactics like `iAssumption`)

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Actual Coq goal (without pretty printing):

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- Hypotheses can be moved to the spatial context Π (by tactics like `iAssumption`)
- Closed under elimination of $\vee$, $\wedge$, $\exists$, $\forall$, ...

Where does the $\Box$ modality come from?

The properties of $\square$ that are needed

- Hypotheses can be duplicated (by tactics like `iSplitL`)
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  $\square\, P \vdash \mathsf{emp}$
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  $\square\, P \vdash P$
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  $\square(\exists x.\, P) \vdash (\exists x.\, \square\, P)$ and $\square(\forall x.\, P) \vdash (\forall x.\, \square\, P)$

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  $\square(\exists x.\, P) \vdash (\exists x.\, \square\, P)$ and $\square(\forall x.\, P) \vdash (\forall x.\, \square\, P)$

Problem: The $\square$ modality cannot be defined in terms of just the BI connectives for some logics (*e.g.* Iris) [Bizjak&Birkedal,MFPS'17]

We will extend BI logics with a new modality

Persistent and intuitionistic propositions

Recall:

	Not droppable	**Droppable**
Not duplicable	Any	Affine $\langle affine \rangle$
Duplicable	Persistent $\boxdot$	Intuitionistic = (affine & persistent) $\square$

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The intuitionistic modality $\square$

- Widely used in practice
- We do not know how to axiomatize it using a small set of axioms

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The intuitionistic modality $\square$

- Widely used in practice
- We do not know how to axiomatize it using a small set of axioms

The persistent modality $\boxdot$

- Can be axiomatized using a small set of axioms
- Can be used to define the $\square$ modality and derive its laws

MoBIs: BIs with a persistence modality

The **persistence modality**:

$$\boxdot\, P \quad \triangleq \quad \text{“}P\text{ holds without ownership of non-exclusive resources”}$$

MoBI $\triangleq$ BI $+ \boxdot +$ additional axioms

MoBIs: BIs with a persistence modality

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MoBI $\triangleq$ BI $+\ \boxdot\ +$ additional axioms

- We can now define:

$$\text{persistent}(P) \triangleq P \vdash \boxdot\, P$$
$$\text{intuitionistic}(P) \triangleq P \vdash \square\, P \qquad\qquad \square\, P \triangleq \langle \mathit{affine} \rangle\, (\boxdot\, P)$$

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- We can now define:

$$\begin{aligned} \text{persistent}(P) &\triangleq P \vdash \boxdot\, P \\ \text{intuitionistic}(P) &\triangleq P \vdash \square\, P \qquad\qquad \square\, P \triangleq \langle \mathit{affine} \rangle (\boxdot\, P) \end{aligned}$$

- The axioms are generalized from Iris's to general (non-affine) BI

$$\frac{P \vdash Q}{\boxdot\, P \vdash \boxdot\, Q} \qquad \begin{array}{c} \boxdot\, P \vdash \boxdot(\boxdot\, P) \\ \text{emp} \vdash \boxdot\, \text{emp} \end{array} \qquad \begin{array}{c} (\forall x.\ \boxdot\, P) \vdash \boxdot\,(\forall x.\ P) \\ \boxdot\,(\exists x.\ P) \vdash (\exists x.\ \boxdot\, P) \end{array} \qquad (\boxdot\, P) * Q \vdash \boxdot\, P$$

What's more in the MoSeL paper [Krebbers *et al.*,ICFP'18]?

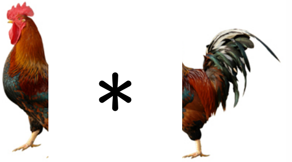
- Instantiations of MoSeL using 5 very different existing logics
  Iris, Fairis, iGPS, CFML, and CHL
- An Instantiation of MoSeL using a new logic
  Based on *ordered resource algebras*—a general model for MoBIs
- Semi-automated tactics using MoSeL for CFML and CHL
  For example, to support read-only permissions in CFML

Thank you!

Download MoSeL at `http://iris-project.org/`

Advertisement. I am currently looking for:

- A PhD student (4 years)
- A post-doc (1 year)



Topics: Separation logics for multilingual programs, asynchronous I/O, non-functional properties, verified compilation, ...

Interested/Know someone? Get in touch!