MoSeL: A General, Extensible Modal Framework for Interactive Proofs in Separation Logic

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Separation logic [O’Hearn, Reynolds, and Yang, 2001]

Propositions $P, Q$ denote ownership of resources

Separating conjunction $P \ast Q$:
The resources consists of separate parts satisfying $P$ and $Q$

Basic example:

$$\{x \mapsto v_1 \ast y \mapsto v_2\} \text{swap}(x, y)\{x \mapsto v_2 \ast y \mapsto v_1\}$$

the $\ast$ ensures that $x$ and $y$ are different memory locations
Why is separation logic useful?

Separation logic is very useful:
▶ It provides a high level of modularity
▶ It scales to fancy PL features like concurrency

Just in Coq, there is an ever growing collection of separation logics:
▶ Bedrock
▶ CFML
▶ Charge!
▶ CHL
▶ FCSL
▶ Iris
▶ VST
▶ …
The challenge

When developing a new separation logic in a proof assistant, one has to:

1. Prove soundness
2. Develop tactics to carry out proofs
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When developing a new separation logic in a proof assistant, one has to:

1. Prove soundness
2. Develop tactics to carry out proofs

These steps are tedious, can we simplify them?
In prior work, we proposed solutions for both problems:

1. Proving soundness: Iris [POPL’15, ICFP’16, ESOP’17, JFP’18]
2. Tactics: Iris Proof Mode [POPL’17]
A general, language-independent, framework for modeling your own domain specific higher-order separation logics
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- **General**: unifies the reasoning principles in many other logics
A general, language-independent, framework for modeling your own domain specific higher-order separation logics

- **General**: unifies the reasoning principles in many other logics
- **Language-independent**: parameterized by the language

Iris [POPL’15, ICFP’16, ESOP’17, JFP’18]
A general, language-independent, framework for **modeling your own domain specific** higher-order separation logics

- **General:** unifies the reasoning principles in many other logics
- **Language-independent:** parameterized by the language
- **Modeling logics:** can be used to model domain specific logics
  - iGPS for weak memory [ECOOP’17]
  - RustBelt’s lifetime logic [POPL’18]
  - ReLoC for program refinements [LICS’18]
  - Iron for resource management [manuscript]
Lemma test \(\{A\}\) (\(\text{P Q : iProp}\)) (\(\text{Ψ : A \rightarrow iProp}\)):
\[
\text{P} \ast (\exists \text{a, Ψ a}) \ast \text{Q} \rightarrow \text{Q} \ast \exists \text{a, P} \ast Ψ \text{a}.
\]

Proof.
- iIntros "[H1 [H2 H3]]".
- iDestruct "H2" as (x) "H2".
- iSplitL "H3".
- iAssumption.
- iExists x.
- iFrame.

Qed.
**Lemma** test \( \{A\} \) (\( P \, Q : \text{iProp} \) ) (\( \Psi : A \rightarrow \text{iProp} \)):

\[
P \ast (\exists \ a, \ \Psi \ a) \ast Q \vdash Q \ast \exists \ a, \ P \ast \Psi \ a.
\]

**Proof.**

\begin{verbatim}
Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
  iSplitL "H3".
  - iAssumption.
  - iExists x.
  - iFrame.
Qed.
\end{verbatim}
Lemma test \{A\} (P Q : iProp) (Ψ : A → iProp) :
  P * (∃ a, Ψ a) * Q →* Q * ∃ a, P * Ψ a.

Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
  iSplitL "H3".
  - iAssumption.
  - iExists x.
  iFrame.
Qed.

1 subgoal
A : Type
P, Q : iProp
Ψ : A → iProp
x : A
____________________________________________________________________(1/1)
"H1" : P
"H2" : Ψ x
"H3" : Q
____________________________________________________________________*
Q * (∃ a : A, P * Ψ a)
Iris Proof Mode [Krebbers et al., POPL’17]: Coq tactics for Iris

Lemma test \{A\} (P Q : iProp) (Ψ : A → iProp):
  P * (∃ a, Ψ a) * Q →* Q * ∃ a, P * Ψ a.

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Q * (∃ a : A, P * Ψ a)

* means: resources should be split
Lemma test \{A\} (P Q : iProp) (Ψ : A → iProp) :
    P ∗ (∃ a, Ψ a) ∗ Q ⊢ Q ∗ ∃ a, P ∗ Ψ a.

Proof.
   iIntros "[H1 [H2 H3]]".
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"H3" : Q
________________________∗
Q ∗ (∃ a : A, P ∗ Ψ a)

* means: resources should be split
Lemma test \{A\} (P Q : iProp) (Ψ : A \rightarrow iProp) :
    P \times (∃ a, Ψ a) \times Q \rightarrow Q \times ∃ a, P \times Ψ a.

Proof.
    iIntros "[H1 [H2 H3]]".
    iDestruct "H2" as (x) "H2".
    iSplitL "H3".
    - iAssumption.
    - iExists x.
      iFrame.
      Qed.

2 subgoals
A : Type
P, Q : iProp
Ψ : A \rightarrow iProp
x : A
______________________________(1/2)
"H3" : Q
______________________________*
Q
______________________________(2/2)
"H1" : P
"H2" : Ψ x
______________________________*
∃ a : A, P \times Ψ a

The hypotheses for the left conjunct
Lemma test \{A\} (P Q : iProp) (Ψ : A → iProp) :
  P * (∃ a, Ψ a) * Q →* Q * ∃ a, P * Ψ a.

Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
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  - iExists x.
  - iFrame.

Qed.
Lemma test \{A\} (P Q : iProp) (Ψ : A → iProp) :
  P * (∃ a, Ψ a) * Q ⊢ Q * ∃ a, P * Ψ a.

Proof.
  iIntros "[H1 [H2 H3]]".
  by iFrame.
Qed.

We can also solve this lemma automatically
The **good things** about Iris Proof Mode

It enabled mechanized proofs in many papers because:

- **Proofs have the look and feel of ordinary Coq proofs**
  For many Coq tactics `tac`, it has a variant `iTac`
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- **Support for advanced features of separation logic**
  Higher-order quantification, invariants, ghost state, later
  ▷ modality, . . .
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- **Integration with tactics for proving programs**
  Symbolic execution tactics for weakest preconditions
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- **Support for advanced features of separation logic**
  Higher-order quantification, invariants, ghost state, later `▷` modality, ...  

- **Integration with tactics for proving programs**
  Symbolic execution tactics for weakest preconditions

- **Usable in practice**
  Used for any project involving Iris today
The bad thing about Iris Proof Mode

The implementation is tied to Iris

WANTED Iris Proof Mode
It sounds easy [Krebbers et al., POPL'17]:

[...] we believe that our proof mode is very generic, and can be applied to a variety of different embedded logics [...]

Making Iris Proof Mode independent of Iris

It sounds easy [Krebbers et al., POPL’17]:

[…] we believe that our proof mode is very generic, and can be applied to a variety of different embedded logics […]

But doing it generally will be more challenging
Problem #1: Iris propositions are affine

In Iris you may “forget” about resources:

\[
\{ \ell_1 \mapsto v_1 \ast \ell_2 \mapsto v_2 \} \ell_2 := ! \ell_1 \{ \ell_2 \mapsto v_1 \}
\]
Problem #1: Iris propositions are affine

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\{ l_1 \mapsto v_1 \ast l_2 \mapsto v_2 \} \ l_2 := ! l_1 \ \{ l_2 \mapsto v_1 \}
\]

Due to the **affinity axiom** \( P \ast Q \vdash Q \), which is hard-wired into many tactics:

- **iClear**
  \[
  \begin{array}{c}
  \Pi \vdash Q \\
  \hline
  \Pi, P \vdash Q
  \end{array}
  \]

- **iAssumption**
  \[
  \begin{array}{c}
  \Pi, P \vdash P \\
  \hline
  \Pi, P \vdash Q
  \end{array}
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Problem #1: Iris propositions are affine

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- **iClear**
  \[
  \Pi \vdash Q \\
  \frac{}{\Pi, P \vdash Q}
  \]

- **iAssumption**
  \[
  \Pi, P \vdash P \\
  \frac{}{\Pi \vdash Q}
  \]

**Not having the affinity axiom is useful:** precise accounting of resources (recall Aleš’s talk)

**Challenge:** How to disentangle the affinity axiom from the Iris tactics?
Problem #2: No tactical support for derived logics

Coq (Prop) \[\rightarrow\] propositions defined in terms of Iris (iProp)

Proof using standard Coq tactics

Proof using Iris tactics
**Problem #2:** No tactical support for derived logics

- **Coq (Prop)**
  - Proof using standard Coq tactics
  - Propositions defined in terms of

- **Iris (iProp)**
  - Proof using Iris tactics
  - Propositions defined in terms of, $iGpsProp \triangleq View \xrightarrow{\text{mon}} iProp$

- **Derived logic (e.g. iGpsProp)**
Problem #2: No tactical support for derived logics

Coq (\texttt{Prop})

\begin{align*}
\text{Derived logic (e.g. } \texttt{iGpsProp}) & \implies \\
\text{propositions defined in terms of} & \\
\text{Iris (\texttt{iProp})} & \implies \\
\text{propositions defined in terms of, } \texttt{iGpsProp } & \triangleq \text{View } \xrightarrow{\text{mon}} \text{iProp} & \end{align*}

Proof using standard Coq tactics

Proof using Iris tactics

Proof using ???
Problem #2: No tactical support for derived logics

\[ \text{Coq (Prop)} \rightarrow \text{Iris (iProp)} \rightarrow \text{Derived logic (e.g. iGpsProp)} \]

- Proof using standard Coq tactics
- Proof using Iris tactics
- Proof using ???

Challenge: How to reason in logics defined in terms of another
Contributions

**MoSeL:** A General, Extensible Modal Framework for Interactive Proofs in Separation Logic in Coq [Krebbers et al., ICFP’18]

**Contributions:**

- MoSeL is parameterized by a general abstraction of separation logic
- MoSeL supports general and affine separation logics, and combinations thereof
- MoSeL supports reasoning in derived separation logics
- MoSeL can be fine-tuned for each logic using type classes

---

\(^2\)Not in the ICFP’18 paper
Contributions

**MoSeL: A General, Extensible Modal Framework for Interactive Proofs in Separation Logic in Coq** [Krebbers et al., ICFP’18]

Contributions:
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- MoSeL supports reasoning in derived separation logics
- MoSeL can be fine-tuned for each logic using type classes

MoSeL is usable in practice: we used it on 6 very different existing separation logics

| CFML | CHL | Fairis | iGPS | Iris | Iron² |

²Not in the ICFP’18 paper
Implementation of tactics in IPM/MoSeL
Lemma test \(\{A\}\) (\(P\ Q : \text{iProp}\) ) (\(\Psi : A \rightarrow \text{iProp}\) ) :
\[P \ast (\exists\ a, \Psi\ a) \ast Q \rightarrow\ Q \ast \exists\ a, P \ast \Psi\ a.\]

Proof.
\begin{itemize}
  \item iIntros "[H1 [H2 H3]]".
  \item iDestruct "H2" as (x) "H2".
  \item iSplitL "H3".
  \begin{itemize}
    \item iAssumption.
    \item iExists x.
    \item iFrame.
  \end{itemize}
\end{itemize}
Qed.
Lemma test \{A\} (P Q : iProp) (Ψ : A \rightarrow iProp) :
\quad P \ast (\exists a, Ψ a) \ast Q \rightarrow Q \ast \exists a, P \ast Ψ a.

Proof.
iIntros "[H1 [H2 H3]]".
iDestruct "H2" as (x) "H2".
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- iAssumption.
- iExists x.
iFrame.
Qed.
Lemma test \{A\} (P Q : iProp) (Ψ : A → iProp) :
    P * (∃ a, Ψ a) * Q →* Q * ∃ a, P * Ψ a.

Proof.
iIntros "[H1 [H2 H3]]".
iDestruct "H2" as (x) "H2".
Set Printing All.

1 subgoal
A : Type
P, Q : iProp
Ψ : A → iProp
x : A
"H1" : P
"H2" : Ψ x
Q * (∃ a : A, P * Ψ a)
Lemma test {A} (P Q : iProp) (Ψ : A → iProp):
  P * (∃ a, Ψ a) * Q ⊢ Q * ∃ a, P * Ψ a.

Proof.
iIntros "[H1 [H2 H3]]".
iDestruct "H2" as (x) "H2".
Set Printing All.
How to embed separation logic entailments into Coq?

Visible goal (with pretty printing):

\[ \vec{x} : \vec{\phi} \quad \text{Variables and pure Coq hypotheses} \]
\[ \Pi \quad \text{Spatial separation logic hypotheses} \]
\[ R \quad \text{Separation logic goal} \]

Actual Coq goal (without pretty printing):

\[ \vec{x} : \vec{\phi} \Pi \models Q \]

Where:

\[ \Pi \models Q \equiv \ast \Pi \vdash Q \]
How to embed separation logic entailments into Coq?

Visible goal (with pretty printing):

\[ \bar{x} : \vec{\phi} \quad \text{Variables and pure Coq hypotheses} \]

\[ \Pi \quad \text{Spatial separation logic hypotheses} \]

\[ R \quad \text{Separation logic goal} \]

Actual Coq goal (without pretty printing):

\[ \bar{x} : \vec{\phi} \]

\[ \Pi \vdash Q \]

Where:

\[ \Pi \vdash Q \triangleq \ast \Pi \vdash Q \]
Implementation of the iSplitL/iSplitR tactic

Tactics implemented by reflection as mere lemmas:

**Lemma** tac_sep_split \(\Pi\ \Pi_1\ \Pi_2\ \text{lr}\ \text{js}\ Q_1\ Q_2\) :

\[
\text{envs}\_\text{split}\ \text{lr}\ \text{js}\ \Pi = \text{Some}\ (\Pi_1,\Pi_2) \rightarrow \\
(\Pi_1 \vdash Q_1) \rightarrow (\Pi_2 \vdash Q_2) \rightarrow \Pi \vdash Q_1 \ast Q_2.
\]

\[
\frac{\Pi_1 \vdash Q_1 \quad \Pi_2 \vdash Q_2}{\Pi_1, \Pi_2 \vdash Q_1 \ast Q_2}
\]
Implementation of the iSplitL/iSplitR tactic

Tactics implemented by reflection as mere lemmas:

\[
\text{Lemma } \text{tac\_sep\_split } \Pi \ \Pi_1 \ \Pi_2 \ \text{l}\text{r} \ \text{js} \ Q_1 \ Q_2 :
\]
\[
\text{envs\_split } \text{l}\text{r} \ \text{js} \ \Pi = \text{Some } (\Pi_1, \Pi_2) \rightarrow
\]
\[
(\Pi_1 \vdash Q_1) \rightarrow (\Pi_2 \vdash Q_2) \rightarrow \Pi \vdash Q_1 \ast Q_2 .
\]

Context splitting implemented as a computable Coq function

\[
\frac{\Pi_1 \vdash Q_1 \quad \Pi_2 \vdash Q_2}{\Pi_1, \Pi_2 \vdash Q_1 \ast Q_2}
\]
Implementation of the iSplitL/iSplitR tactic

Tactics implemented by reflection as mere lemmas:

Lemma tac_sep_split \(\Pi_1 \Pi_2\) lr js \(Q_1\) \(Q_2\) :
\[
\text{envs_split lr js } \Pi = \text{Some} (\Pi_1,\Pi_2) \rightarrow
(\Pi_1 \vdash Q_1) \rightarrow (\Pi_2 \vdash Q_2) \rightarrow \Pi \vdash Q_1 \ast Q_2.
\]

Context splitting implemented as a computable Coq function

Ltac wrappers around the reflective tactic:

Tactic Notation "iSplitL" constr(Hs) :=
  let Hs := words Hs in
  let Hs := eval vm_compute in (INamed <$> Hs) in
  eapply tac_sep_split with _ _ Left Hs _ _;
  [pm_reflexivity ||
   fail "iSplitL: hypotheses" Hs "not found"
   | (* goal 1 *)
   | (* goal 2 *) ].
Implementation of the iSplitL/iSplitR tactic

Tactics implemented by reflection as mere lemmas:

**Lemma** tac_sep_split \( \Pi \Pi_1 \Pi_2 \text{ lr js } Q_1 Q_2 : \)
\[
envs\_split \text{ lr js } \Pi = \text{ Some } (\Pi_1,\Pi_2) \rightarrow \\
(\Pi_1 \vdash Q_1) \rightarrow (\Pi_2 \vdash Q_2) \rightarrow \Pi \vdash Q_1 * Q_2.
\]

Context splitting implemented as a computable Coq function

**Ltac wrappers around the reflective tactic:**

**Tactic Notation** "iSplitL" constr(Hs) :=
\[
\text{let } Hs := \text{ words Hs in} \\
\text{let } Hs := \text{ eval vm\_compute in (INamed } <\$$> Hs) in \\
\text{eapply tac\_sep\_split with } _ _ \text{ Left Hs } _ _ ; \\
\text{[pm\_reflexivity } || \\
\text{fail "iSplitL: hypotheses" Hs "not found"} \\
\text{| (* goal 1 *)} \\
\text{| (* goal 2 *) ]}.
\]

Report sensible error to the user
Making MoSeL separation logic independent

- Instance of a separation logic abstraction
- Type class instances to fine tune tactics

MoSeL’s tactics
Making MoSeL separation logic independent

Instance of a separation logic abstraction

Type class instances to fine tune tactics

MoSeL’s tactics
How to abstract over separation logics?
BI logics: An abstract interface for separation logics

A Bunched Implications (BI) logic [O’Hearn&Pym,99] is a preorder \((Prop, ⊧)\) with:

- Operations True, False, \(\land, \lor, \Rightarrow, \forall, \exists\) satisfying the axioms of intuitionistic logic
- Operations \(\text{emp}, *, \neg\) * satisfying:

\[
\begin{align*}
\text{emp} * P & \vdash P \\
P * Q & \vdash Q * P \\
(P * Q) * R & \vdash P * (Q * R)
\end{align*}
\]

\[
\begin{align*}
P_1 \vdash Q_1 & \quad P_2 \vdash Q_2 \\
\frac{P_1 \vdash Q_1 \quad P_2 \vdash Q_2}{P_1 * P_2 \vdash Q_1 * Q_2} \\
\frac{P_1 \vdash Q_1 \quad P_2 \vdash Q_2}{P_1 * P_2 \vdash Q_1 * Q_2}
\end{align*}
\]

\[
\begin{align*}
P \vdash Q & \neg\neg R \\
\frac{P_1 \vdash Q_1 \quad P_2 \vdash Q_2}{P_1 * P_2 \vdash Q_1 * Q_2} \\
\frac{P_1 \vdash Q_1 \quad P_2 \vdash Q_2}{P_1 * P_2 \vdash Q_1 * Q_2}
\end{align*}
\]
BI logics: An abstract interface for separation logics

A **Bunched Implications (BI) logic** [O’Hearn&Pym,99] is a preorder \((\text{Prop}, \vdash)\) with:

- **Operations** `True, False, \land, \lor, \Rightarrow, \forall, \exists` satisfying the axioms of intuitionistic logic
- **Operations** `emp, *, \rightarrowstar` satisfying:

\[
\begin{align*}
\text{emp} \star P & \not\vdash P \\
P \star Q & \vdash Q \star P \\
(P \star Q) \star R & \vdash P \star (Q \star R)
\end{align*}
\]

\[
\begin{align*}
P_1 \vdash Q_1 & \quad P_2 \vdash Q_2 \quad P \star Q \vdash R \\
P_1 \star P_2 & \vdash Q_1 \star Q_2 \\
P & \vdash Q \rightarrowstar R
\end{align*}
\]

**Structure** `bi := Bi {` 

`bi_car :> Type;`

`bi_entails : bi_car \rightarrow bi_car \rightarrow \text{Prop};`

`bi_forall : \forall A, (A \rightarrow bi_car) \rightarrow bi_car;`

`bi_sep : bi_car \rightarrow bi_car \rightarrow bi_car;`

`(* other separation logic operators and axioms *)` 

`}`.
Proofs in MoSeL

Proofs in a specific logic:

Lemma test \{A\} (P Q : iGpsProp) (Ψ : A → iGpsProp):
  P * (∃ a, Ψ a) * Q →* Q * ∃ a, P * Ψ a.

Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
  iSplitL "H3".
  - iAssumption.
  - iExists x.
    iFrame.
Qed.

Proofs for all logics:

Lemma test \{PROP : bi\} \{A\} (P Q : PROP) (Ψ : A → PROP):
  P * (∃ a, Ψ a) * Q →* Q * ∃ a, P * Ψ a.

Proof.
  iIntros "[H1 [H2 H3]]".
  iDestruct "H2" as (x) "H2".
  iSplitL "H3".
  - iAssumption.
  - iExists x.
    iFrame.
Qed.
Proofs in MoSeL

Proofs in a specific logic:

Lemma test \( \{A\} (P \land Q : \text{iGpsProp})(\Psi : A \rightarrow \text{iGpsProp}) : P \land (\exists a, \Psi a) \land Q \rightarrow Q \land \exists a, P \land \Psi a. \)

Proof.

\(\begin{align*}
&\text{iIntros "[H1 [H2 H3]]".} \\
&\text{iDestruct "H2" as} (x) "H2". \\
&\text{iSplitL "H3".} \\
&\text{iAssumption.} \\
&\text{− iExists x.} \\
&\text{iFrame.} \\
&\text{Qed.}
\end{align*}\)

Proofs for all logics:

Lemma test \( \{\text{PROP : bi}\} \{A\} (P \land Q : \text{PROP})(\Psi : A \rightarrow \text{PROP}) : P \land (\exists a, \Psi a) \land Q \rightarrow Q \land \exists a, P \land \Psi a. \)

Proof.

\(\begin{align*}
&\text{iIntros "[H1 [H2 H3]]".} \\
&\text{Destruct "H2" as} (x) "H2". \\
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&\text{− iExists x.} \\
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&\text{Qed.}
\end{align*}\)

Lemma for another logic than Iris

Lemma universally quantified in the BI logic
Proofs in MoSeL

Proofs in a specific logic:

Lemma test \(\{A\}\) \((P Q : \text{iGpsProp})\) \((\Psi : A \rightarrow \text{iGpsProp})\):
\[ P \ast (\exists a, \Psi a) \ast Q \rightarrow Q \ast \exists a, P \ast \Psi a. \]
Proof.
  iIntros "[H1 [H2 H3]]".
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Qed.

Proofs for all logics:

Lemma test \(\{\text{PROP} : \text{bi}\}\) \(\{A\}\) \((P Q : \text{PROP})\) \((\Psi : A \rightarrow \text{PROP})\):
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Qed.

Lemma for another logic than Iris

Lemma universally quantified in the BI logic
Addressing challenge #1: Disentangling the affinity axiom

$P \ast Q \vdash Q$
A poor man’s solution

Make two versions of the tactics

1. For affine logics (like Iris and iGPS)
2. For non-affine logics (like CFML and CHL)
A poor man’s solution

Make two versions of the tactics

1. For affine logics (like Iris and iGPS)
2. For non-affine logics (like CFML and CHL)

Problems:

- Duplicate work/maintenance
- Some logics mix affine and non-affine propositions, for example:

  \[
  \begin{align*}
  \text{GC locations} \quad \text{(affine)} & \quad \text{Non-GC locations} \quad \text{(not affine)} \\
  \ell \mapsto_{\text{GC}} v & \quad \ell \mapsto v
  \end{align*}
  \]

  (Another example in [Tassarotti et al., ESOP’17])
Key idea

► **Don’t:** classify whether the whole logic is affine
► **Do:** classify whether individual propositions are affine
Classifying whether propositions are affine

Affine propositions:

\[
\text{affine}(P) \triangleq P \vdash \text{emp}
\]
(propositions that can be "thrown away")

The new tactics:

\[
\text{iClear} \\
\Pi \vdash Q \quad \text{affine}(P) \\
\Pi, P \vdash Q
\]

\[
\text{iAssumption} \\
\text{affine}(\Pi) \\
\Pi, Q \vdash Q
\]
Classifying whether propositions are affine in Coq

A new type class:

\[
\text{Class Affine } \{\text{PROP : bi}\} (Q : \text{PROP}) := \text{affine } : Q \vdash \text{emp}.
\]

Instances:

- Tell MoSeL that specific connectives are affine:

  Instance mapsto_gC_affine l v : Affine (l \mapsto_{gC} v).

- Capture that affine propositions are closed under most connectives:

  Instance sep_affine \{\text{PROP : bi}\} (P Q : \text{bi}) :
  \text{Affine } P \rightarrow \text{Affine } Q \rightarrow \text{Affine } (P \ast Q).
MoSeL: A General, Extensible Modal Framework for Interactive Proofs in Separation Logic in Coq

What about modalities?
The affine modality

**The affine modality:**

\[ \langle \text{affine} \rangle P \triangleq P \land \text{emp} \]

\[ \approx \text{“} P \text{ holds using just affine resources”} \]

This modality is useful for reasoning about affinity
The affine modality

The **affine modality**:

\[
\langle \text{affine} \rangle P \triangleq P \land \text{emp}
\]

\[
\approx \quad "P \text{ holds using just affine resources}"
\]

This modality is useful for reasoning about affinity

- Can be used to turn any proposition into an affine version, *e.g.*
  A wand that can be dropped \( \langle \text{affine} \rangle (P \rightarrow Q) \)
The affine modality

The **affine modality**:

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This modality is useful for reasoning about affinity

- Can be used to turn any proposition into an affine version, *e.g.*
  A wand that can be dropped \( \langle \text{affine} \rangle (P \rightarrow Q) \)

- Commutes with most operators, *e.g.*
  \( \langle \text{affine} \rangle (P \lor Q) \dashv \vdash \langle \text{affine} \rangle P \lor \langle \text{affine} \rangle Q \)
The affine modality

The affine modality:

\[ \langle \text{affine} \rangle P \triangleq P \land \text{emp} \]
\[ \approx \text{“} P \text{ holds using just affine resources”} \]

This modality is useful for reasoning about affinity

- Can be used to turn any proposition into an affine version, e.g.
  A wand that can be dropped \( \langle \text{affine} \rangle (P \rightarrow Q) \)

- Commutes with most operators, e.g.
  \[ \langle \text{affine} \rangle (P \lor Q) \dashv \vdash \langle \text{affine} \rangle P \lor \langle \text{affine} \rangle Q \]

- Gives rise to an alternative classification of affine propositions
  \( \text{affine}(P) \iff P \vdash \langle \text{affine} \rangle P \)
The idea of carving out classes of propositions and defining their corresponding modalities is widely applicable:

- Persistent propositions
- Intuitionistic propositions
- Absorbing propositions \(\langle \text{absorb} \rangle\)
- Timeless propositions (in step-indexed logics) \(\llcorner, \lrcorner\)
- Objective propositions (in iGPS) \(\langle \text{obj} \rangle, \langle \text{subj} \rangle\)
- Normal propositions (in CFML) \(\langle \text{normal} \rangle\)
- ...

The paper shows how to modularly deal with such classes and use them in general tactics.
The idea of carving out classes of propositions and defining their corresponding modalities is widely applicable:

- Persistent propositions
- Intuitionistic propositions
- Absorbing propositions
- Timeless propositions (in step-indexed logics)
- Objective propositions (in iGPS)
- Normal propositions (in CFML)
- ...

The paper shows how to modularly deal with such classes and use them in general tactics, we discuss two
### Persistent and intuitionistic propositions

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## Persistent and intuitionistic propositions

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### Intuitionistic propositions:
- Widely used in practice, *e.g.* for reasoning about persistent (and garbage collected) data structures
- MoSeL has special support for them
Lemma test \{PROP : bi\} \{A\}

\( (P \ Q : PROP) (\Psi : A \rightarrow PROP) : \)
\( P \ast \square (\exists \ a, \Psi \ a) \rightarrow \exists \ a, \Psi \ a \ast (P \ast \Psi \ a). \)

Proof.

iIntros "[H1 \#H2]."

iDestruct "H2" as (x) "H2".

iExists x.

iSplitL "H2".

– iAssumption.

– by iFrame.

Qed.
Lemma test \{PROP : bi\} \{A\}

$(P \ A \rightarrow \text{PROP}) :$

$P \ A \rightarrow (\exists a, \Psi a) \rightarrow (\exists a, \Psi a) \rightarrow \Psi a \rightarrow (P \Psi a).$

Proof.

iIntros "\[H1 \#H2\]."

iDestruct "H2" as \(x\) "H2".

iExists x.

iSplitL "H2".

iAssumption.

by iFrame.

Qed.
Intuitionistic propositions in MoSeL

Lemma test {PROP : bi} {A}

\[(P \land Q : PROP) (\Psi : A \rightarrow PROP) : P \land \Box (\exists a, \Psi a) \leftrightarrow \exists a, \Psi a \land (P \land \Psi a).\]

Proof.

iIntros "[H1 #H2]."
iDestruct "H2" as (x) "H2".
iExists x.
iSplitL "H2".
- iAssumption.
- by iFrame.

Qed.

Moves hypothesis to **intuitionistic** context
Lemma test \{PROP : bi\} \{A\}

\((P, Q : \text{PROP}) (\Psi : A \rightarrow \text{PROP}) : 
P \ast \Box (\exists a, \Psi a) \rightarrow \exists a, \Psi a \ast (P \ast \Psi a)\).

Proof

iIntros "[H1 #H2]".
iDestruct "H2" as (x) "H2".
iExists x.
iSplitL "H2".
  - iAssumption.
  - by iFrame.
Qed.
Lemma test \{PROP : bi\} \{A\}

\((P \land Q : PROP) (\Psi : A \to PROP) :\)

\(P \land \Box (\exists a, \Psi a) \to \exists a, \Psi a \land (P \land \Psi a).\)

Proof.

iIntros "[H1 \#H2]".
iDestruct "H2" as (x) "H2".
iExists x.

iSplitL "H2".

- iAssumption.

Do not need to split intuitionistic context.
Lemma test \{PROP : bi\} \{A\}

\[(P \land Q : PROP) (\Psi : A \rightarrow PROP) :\]
\[P \ast \Box (\exists a, \Psi a) \rightarrow \exists a, \Psi a \ast (P \ast \Psi a).\]

Proof.
\begin{itemize}
  \item iIntros "[H1 #H2]".
  \item iDestruct "H2" as (x) "H2".
  \item iExists x.
  \item iSplitL "H2".
\end{itemize}

\begin{itemize}
  \item iAssumption.
  \item by iFrame.
\end{itemize}

Qed.

2 subgoals
PROP : bi
A : Type
P, Q : PROP
\Psi : A \rightarrow PROP
x : A

\begin{itemize}
  \item \[\Psi x\] \hspace{2cm} (1/2)
  \item \[\Psi x\] \hspace{2cm} (2/2)
\end{itemize}
Formal treatment of the intuitionistic context

Visible Coq goal (with pretty printing):

\[ \Gamma, \Pi \vdash Q \equiv (\forall \Gamma \cdot \ast (\ast \Pi)) \vdash Q \]

Properties of the intuitionistic context \( \Gamma \):

▶ Hypotheses can be duplicated (by tactics like \( \text{iSplitL} \))
▶ Hypotheses can be dropped (by tactics like \( \text{iClear} \) and \( \text{iAssumption} \))
▶ Hypotheses can be moved to the spatial context \( \Pi \) (by tactics like \( \text{iAssumption} \))
▶ Closed under elimination of \( \lor \), \( \land \), \( \exists \), \( \forall \), \ldots
Formal treatment of the intuitionistic context

Visible Coq goal (with pretty printing):

\[ \Gamma \text{ Intuitionistic separation logic hypotheses} \]
\[ \Pi \text{ Spatial separation logic hypotheses} \]
\[ R \text{ Separation logic goal} \]

Actual Coq goal (without pretty printing):

\[ \Gamma; \Pi \vdash Q \triangleq \square (\land \Gamma) * (\star \Pi) \vdash Q \]
Formal treatment of the intuitionistic context

Visible Coq goal (with pretty printing):

\[ \Gamma \quad \text{Intuitionistic separation logic hypotheses} \]
\[ \Pi \quad \text{Spatial separation logic hypotheses} \]
\[ R \quad \text{Separation logic goal} \]

Actual Coq goal (without pretty printing):

\[ \Gamma; \Pi \vdash Q \triangleq \Box (\bigwedge \Gamma)^* (\star \Pi) \vdash Q \]

Properties of the intuitionistic context \( \Gamma \):

- Hypotheses can be duplicated (by tactics like \texttt{iSPLITL})
Formal treatment of the intuitionistic context

Visible Coq goal (with pretty printing):

\[
\Gamma \text{ Intuitionistic separation logic hypotheses} \\
\Pi \text{ Spatial separation logic hypotheses} \\
R \text{ Separation logic goal}
\]

Actual Coq goal (without pretty printing):

\[
\Gamma; \Pi \vdash Q \triangleq \Box \left( \land \Gamma \right)^* \left( \star \Pi \right) \vdash Q
\]

Properties of the **intuitionistic context** \( \Gamma \):

- Hypotheses can be duplicated (by tactics like `iSplitL`)
- Hypotheses can be dropped (by tactics like `iClear` and `iAssumption`)
Formal treatment of the intuitionistic context

Visible Coq goal (with pretty printing):

\[ \Gamma \quad \text{Intuitionistic separation logic hypotheses} \]
\[ \Pi \quad \text{Spatial separation logic hypotheses} \]
\[ R \quad \text{Separation logic goal} \]

Actual Coq goal (without pretty printing):

\[ \Gamma ; \Pi \models Q \iff \square (\bigwedge \Gamma) \ast (\star \Pi) \vdash Q \]

Properties of the \textit{intuitionistic context} \( \Gamma \):

- Hypotheses can be duplicated (by tactics like \texttt{iSplitL})
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Formal treatment of the intuitionistic context

Visible Coq goal (with pretty printing):

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\Gamma & \quad \text{Intuitionistic separation logic hypotheses} \\
\Pi & \quad \text{Spatial separation logic hypotheses} \\
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\end{align*} \]

Actual Coq goal (without pretty printing):

\[ \Gamma; \Pi \models Q \iff \Box (\bigwedge \Gamma) * (\bigstar \Pi) \vdash Q \]

Properties of the intuitionistic context \( \Gamma \):

- Hypotheses can be duplicated (by tactics like \text{iSplitL})
- Hypotheses can be dropped (by tactics like \text{iClear} and \text{iAssumption})
- Hypotheses can be moved to the spatial context \( \Pi \) (by tactics like \text{iAssumption})
- Closed under elimination of \( \lor, \land, \exists, \forall, \ldots \)
Where does the □ modality come from?
The properties of $\square$ that are needed

- Hypotheses can be duplicated (by tactics like iSplitL)
- Hypotheses can be dropped (by tactics like iClear and iAssumption)
- Hypotheses can be moved to the spatial context (by tactics like iAssumption)
- Closed under elimination of $\lor$, $\land$, $\exists$, $\forall$, ...
The properties of $\□$ that are needed

- Hypotheses can be duplicated (by tactics like iSplitL)
  $$\□ P \vdash \□ P \land \□ P$$

- Hypotheses can be dropped (by tactics like iClear and iAssumption)

- Hypotheses can be moved to the spatial context (by tactics like iAssumption)

- Closed under elimination of $\lor, \land, \exists, \forall, \ldots$
The properties of □ that are needed

▶ Hypotheses can be duplicated (by tactics like iSplitL)
   □P ⊢ □P ⊗ □P

▶ Hypotheses can be dropped (by tactics like iClear and iAssumption)
   □P ⊢ emp

▶ Hypotheses can be moved to the spatial context (by tactics like iAssumption)

▶ Closed under elimination of ∨, ∧, ∃, ∀, …
The properties of $\Box$ that are needed

- Hypotheses can be duplicated (by tactics like $i$SplitL)
  $\Box P \vdash \Box P \ast \Box P$

- Hypotheses can be dropped (by tactics like $i$Clear and $i$Assumption)
  $\Box P \vdash \text{emp}$

- Hypotheses can be moved to the spatial context (by tactics like $i$Assumption)
  $\Box P \vdash P$

- Closed under elimination of $\lor$, $\land$, $\exists$, $\forall$, ...
The properties of □ that are needed

- Hypotheses can be duplicated (by tactics like iSplitL)
  \[ \square P \vdash_{\square} \square P \ast \square P \]

- Hypotheses can be dropped (by tactics like iClear and iAssumption)
  \[ \square P \vdash \text{emp} \]

- Hypotheses can be moved to the spatial context (by tactics like iAssumption)
  \[ \square P \vdash P \]

- Closed under elimination of \( \lor, \land, \exists, \forall, \ldots \)
  \[ \square(\exists x. P) \vdash (\exists x. \square P) \] and \[ \square(\forall x. P) \vdash (\forall x. \square P) \]
The properties of □ that are needed

- Hypotheses can be duplicated (by tactics like iSplitL)
  \[\Box P \vdash \Box P \ast \Box P\]

- Hypotheses can be dropped (by tactics like iClear and iAssumption)
  \[\Box P \vdash \text{emp}\]

- Hypotheses can be moved to the spatial context (by tactics like iAssumption)
  \[\Box P \vdash P\]

- Closed under elimination of \(\lor, \land, \exists, \forall, \ldots\)
  \[\Box(\exists x. P) \vdash (\exists x. \Box P)\] and \[\Box(\forall x. P) \vdash (\forall x. \Box P)\]

**Problem:** The □ modality cannot be defined in terms of just the BI connectives for some logics (e.g. Iris) [Bizjak&Birkedal, MFPS'17]
We will extend BI logics with a new modality
## Persistent and intuitionistic propositions

**Recall:**

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- Intuitionistic modality is widely used in practice.
- We do not know how to axiomatize it using a small set of axioms.
- Persistent modality can be axiomatized using a small set of axioms.
- It can be used to define the modality and derive its laws.
Persistent and intuitionistic propositions

Recall:

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The intuitionistic modality □

- Widely used in practice
- We do not know how to axiomatize it using a small set of axioms
Persistent and intuitionistic propositions

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The intuitionistic modality □

- Widely used in practice
- We do not know how to axiomatize it using a small set of axioms

The persistent modality □

- Can be axiomatized using a small set of axioms
- Can be used to define the □ modality and derive its laws
MoBIs: BI$\lambda$s with a persistence modality

The persistence modality:

\[ \square P \triangleq "P \text{ holds without ownership of non-exclusive resources}" \]

\[ \text{MoBI} \triangleq \text{BI} + \square + \text{additional axioms} \]
MoBIs: BIs with a persistence modality

The **persistence modality**:

\[ \Box P \triangleq \text{"} P \text{ holds without ownership of non-exclusive resources"} \]

\[ \text{MoBI} \triangleq \text{BI} + \Box \text{ + additional axioms} \]

We can now define:

\[ \text{persistent}(P) \triangleq P \vdash \Box P \]
\[ \text{intuitionistic}(P) \triangleq P \vdash \Box P \]
\[ \Box P \triangleq \langle \text{affine} \rangle (\Box P) \]
MoBIs: BIs with a persistence modality

The persistence modality:

\[ \Box P \triangleq "P \text{ holds without ownership of non-exclusive resources}" \]

\[ \text{MoBI} \triangleq \text{BI} + \Box + \text{additional axioms} \]

We can now define:

\[ \text{persistent}(P) \triangleq P \vdash \Box P \]

\[ \text{intuitionistic}(P) \triangleq P \vdash \Box P \]

\[ \Box P \triangleq \langle \text{affine} \rangle (\Box P) \]

The axioms are generalized from Iris’s to general (non-affine) BI

\[ \frac{P \vdash Q}{\Box P \vdash \Box Q} \]

\[ \frac{\Box P \vdash \Box (\Box P)}{\Box P \vdash \Box (\forall x. P)} \]

\[ \frac{(\forall x. \Box P) \vdash \Box (\forall x. P)}{\Box (\exists x. P) \vdash (\exists x. \Box P)} \]

\[ (\Box P) * Q \vdash \Box P \]
What’s more in the MoSeL paper [Krebbers et al., ICFP’18]?

- Instantiations of MoSeL using 5 very different existing logics: Iris, Fairis, iGPS, CFML, and CHL
- An Instantiation of MoSeL using a new logic: Based on ordered resource algebras—a general model for MoBIs
- Semi-automated tactics using MoSeL for CFML and CHL: For example, to support read-only permissions in CFML
Thank you!

Download MoSeL at http://iris-project.org/

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- A post-doc (1 year)

Topics: Separation logics for multilingual programs, asynchronous I/O, non-functional properties, verified compilation, ...

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