An Operational and Axiomatic Semantics for Non-determinism and Sequence Points in C

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What is this program supposed to do?

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  int y = (x = 3) + (x = 4);
  printf("%d %d\n", x, y);
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This program violates the sequence point restriction

- due to two unsequenced writes to x
- resulting in undefined behavior
- thus both compilers are right

Undefined behavior in C

"Garbage in, garbage out" principle

- Programs with undefined behavior are not statically excluded
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A compiler independent C semantics should account for undefined behavior

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while ((x = getchar()) != EOF)
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- Interleaving of subexpressions is possible, for example

```
printf("a") + (printf("b") + printf("c"));
```

  may print "bac"

Contribution

A compiler independent small step operational, and axiomatic, semantics for non-determinism and sequence points, supporting:

- expressions with function calls, assignments, conditionals
- undefined behavior due to integer overflow
- parametrized by integer types
- dynamically allocated memory (`malloc` and `free`)
- non-local control (`return` and `goto`)
- local variables (and pointers to those)
- mutual recursion
- separation logic
- soundness proof fully checked by Coq

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Key idea

Observation: non-determinism corresponds to concurrency
Idea: use the separation logic rule for parallel composition

$$\frac{\{P_1\}\, e_1\, \{Q_1\} \qquad \{P_2\}\, e_2\, \{Q_2\}}{\{P_1 * P_2\}\, e_1 \circledcirc e_2\, \{Q_1 * Q_2\}}$$

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- Split the memory into two disjoint parts
- Prove that e_1 and e_2 can be executed safely in their part
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Disjointness $\Rightarrow$ no sequence point violation

Definition of the memory

Given a set of permissions P, we define:

$$m \in \text{mem} := \text{index} \rightarrow_{\text{fin}} (\text{val} \times P)$$

$$b \in \text{index} := \mathbb{N}$$

$$v \in \text{val} ::= \text{indet} \mid \text{int}_{\tau}\, n \mid \text{ptr}\, b \mid \text{NULL}$$

Integer types: unsigned char, signed int, ...

Permission systems

Actual permissions contains:

- Locked flag to catch sequence point violations
  - Assignment: lock memory location
  - Sequence point: unlock memory locations
- A fraction $(0,1]_{\mathbb{Q}}$ to allow sharing
- Block scope variable or allocated with `malloc` flag

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A permission system P abstracts from these details.

- $\cup, \setminus \; : P \to P \to P$
- $\perp, \subseteq \; : P \to P \to \mathrm{Prop}$
- $\mathrm{kind} : P \to \{\mathrm{Free}, \mathrm{Write}, \mathrm{Read}, \mathrm{Locked}\}$
- $\mathrm{lock}, \mathrm{unlock} : P \to P$
- satisfying certain axioms

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We lift these operations to memories $\mathrm{index} \to_{\mathrm{fin}} (\mathrm{val} \times P)$

The language

Our language

$$
\begin{aligned}
\circledcirc \in \mathsf{binop} &::= \texttt{==} \mid \texttt{<=} \mid \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{/} \mid \texttt{\%} \mid \ldots \\
e \in \mathsf{expr} &::= x_i \mid [v]_\Omega \mid e_1 := e_2 \mid f(\vec{e}) \mid \texttt{load}\ e \mid \texttt{alloc} \\
&\quad\mid \texttt{free}\ e \mid e_1 \circledcirc e_2 \mid e_1 \mathbin{?} e_2 : e_3 \mid (\tau)\, e \\
s \in \mathsf{stmt} &::= e \mid \texttt{skip} \mid \texttt{goto}\ l \mid \texttt{return}\ e \mid \texttt{block}\ c\ s \mid s_1 \,;\, s_2 \\
&\quad\mid l : s \mid \texttt{while}(e)\ s \mid \texttt{if}\ (e)\ s_1\ \texttt{else}\ s_2
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Values $[v]_\Omega$ carry a set Ω of indexes (memory locations) to be unlocked at the next sequence point

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Head reduction for expressions $(e, m) \rightharpoonup_{\mathsf{h}} (e', m')$ (9 rules)

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Small step reduction $\mathbf{S}(k,\ \phi,\ m) \rightharpoonup \mathbf{S}(k',\ \phi',\ m')$ (33 rules)

- (k, ϕ) gives the position in the *whole* program
- Uses evaluation contexts to lift head reduction
- Different ϕs for expressions, statements, function calls

Assertions of separation logic

Defined using a shallow embedding:

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(with e_1 and e_2 side-effect free)

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$$(P \triangleright)\,\rho\, m := P\,\rho\,(\text{unlock}\,(\text{locks}\; m) m)$$

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The Hoare "triples"

Statement judgment $\Delta; J; R \vdash \{P\}\, s\, \{Q\}$

Function conditions

Goto/return conditions

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Function conditions | Goto/return conditions | Q : assert

Expression judgment $\Delta \vdash \{P\}\, e\, \{Q\}$

Q : val $\to$ assert

In the semantics: if P holds beforehand, then

- e does not crash
- $Q\, v$ holds afterwards when terminating with v
- with framing memories that can change at each step

The axiomatic semantics

24 separation logic proof rules, *e.g.*:

For assignments:

$$\frac{\mathrm{Write} \subseteq \mathrm{kind}\ \gamma \quad \{P_1\}\, e_1\, \{Q_1\} \quad \{P_2\}\, e_2\, \{Q_2\} \quad \forall a v\,.\,((Q_1\ a * Q_2\ v) \to ((a \stackrel{\gamma}{\mapsto} -) * R\ a\ v))}{\{P_1 * P_2\}\, e_1 := e_2\, \{\lambda v\,.\,\exists a\,.\,(a \stackrel{\mathrm{lock}\ \gamma}{\mapsto} v) * R\ a\ v\}}$$

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Common separation logic and more complex rules can be derived

Formalization in Coq

- Based on [Krebbers/Wiedijk, FoSSaCS'13]
- Extremely useful for debugging
- Notations close to those on paper
- Various extensions of the separation logic
- Uses lots of automation
- 10 000 lines of code

```
Lemma ax_load Δ A γ e P Q :
  perm_kind γ ≠ Locked →
  Δ\ A ⊨ₑ {{ P }} e {{ λ a, ∃ v, Q a v ∗ valc a ↦{γ} valc v }} →
  Δ\ A ⊨ₑ {{ P }} load e {{ λ v, ∃ a, Q a v ∗ valc a ↦{γ} valc v }}.
```

Future research

- Integration with the C type system [Krebbers, CPP'13]
- Integration with non-aliasing restrictions [Krebbers, CPP'13]
- Interpreter in Coq
- Verification condition generator in Coq
- Automation of separation logic

Questions

Sources: `http://robbertkrebbers.nl/research/ch2o/`