Separation Logic for Non-local Control Flow and Block Scope Variables

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Joint work with Freek Wiedijk

Radboud University Nijmegen

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What is this program supposed to do?

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int *p = NULL;
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} else {
  int j = 10;
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}
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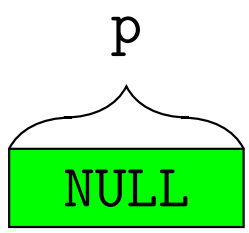

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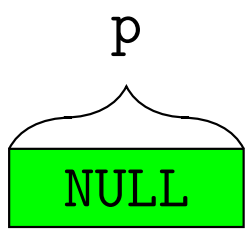

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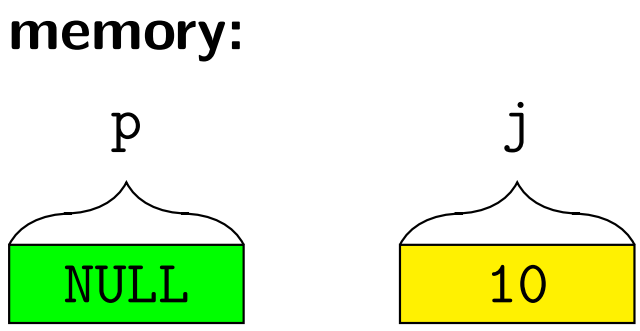

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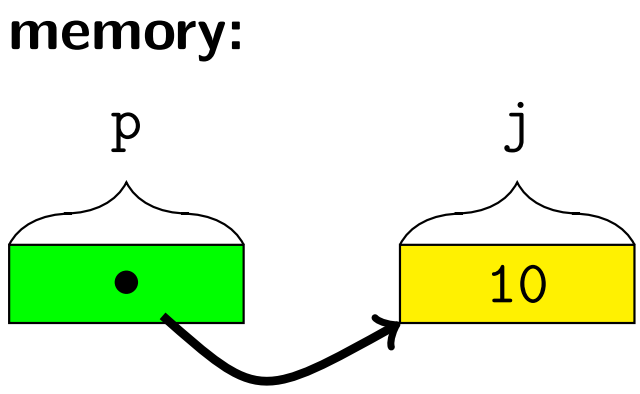

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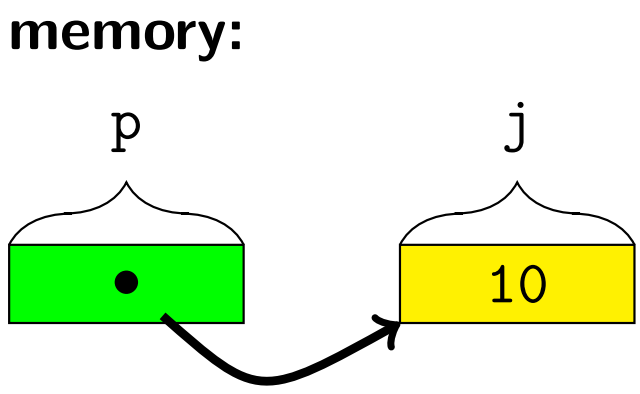

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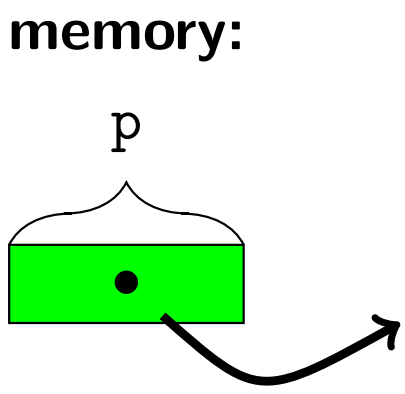

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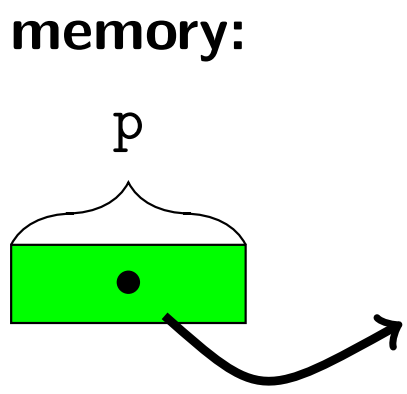

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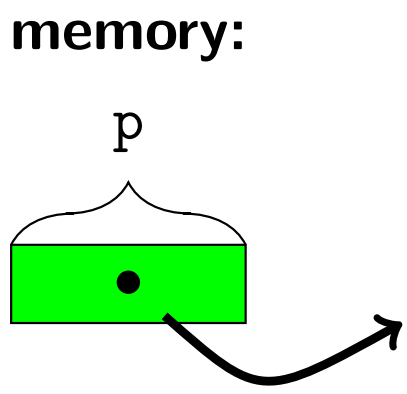

It exhibits undefined behavior, thus it may do *anything*

Why is catching undefined behavior important

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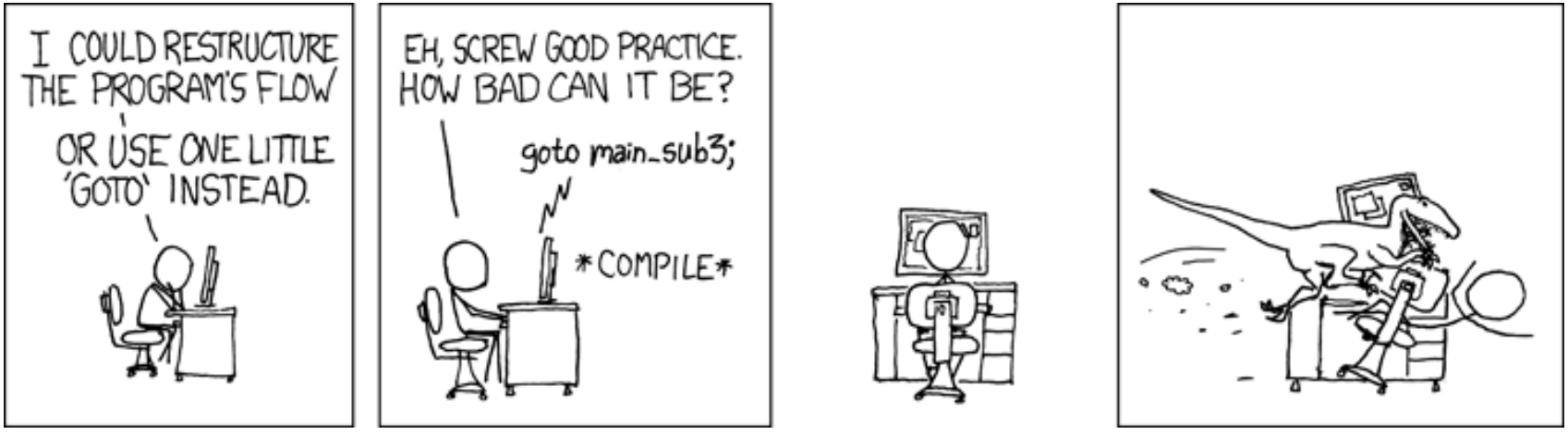
  - whereas they may crash when compiled with an actual compiler

Goto considered harmful?



http://xkcd.com/292/

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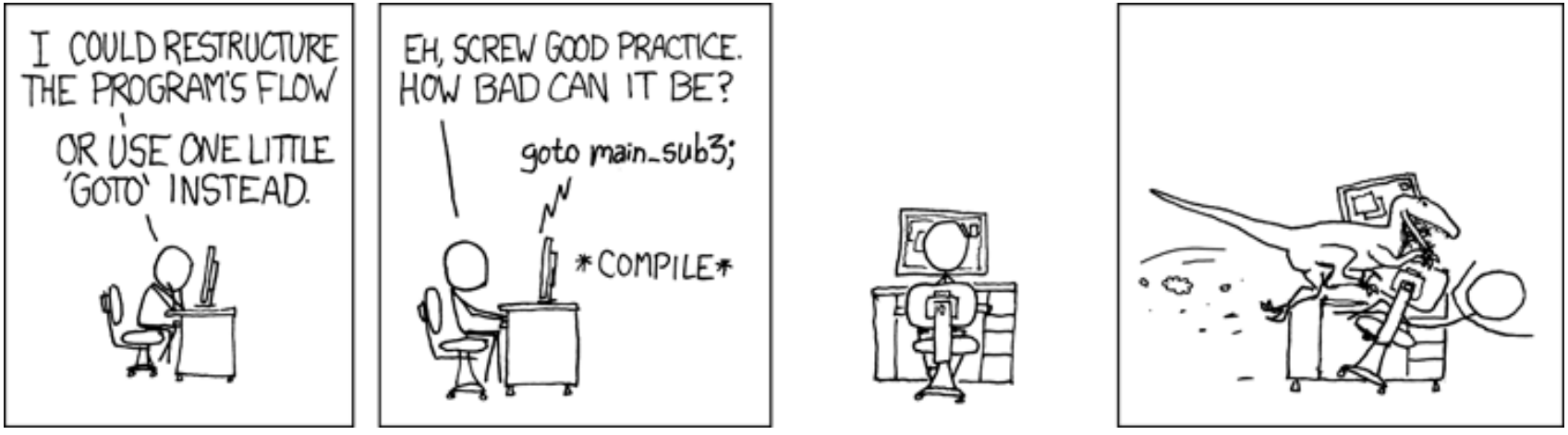

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Not necessarily:

 $\vdash \{P\} \ldots$ `goto main_sub3;` $\ldots \{Q\}$

Why goto

Goto can be useful

- Breaking from multiple nested loops

```
for (int i = 0; i++; i < n) {
  // do something here

  for (int j = i; j++; j < m) {
    // do some work here

    goto outer;
  }
}
outer:;
```

Why goto

Goto can be useful

- Breaking from multiple nested loops
- Systematically cleaning up resources after errors

```
if (!openDataFile())
  goto quit;
if (!getDataFromFile())
  goto closeFileAndQuit;
if (!allocateSomeResources)
  goto freeResourcesAndQuit;

// do actual work here

freeResourcesAndQuit: // free resources
closeFileAndQuit: // close file
quit: // quit!
```

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Goto is used in practice

- The Linux kernel contains about $\sim$ 100.000 uses of goto

```
$ grep -w -r goto ~/src/linux-3.5.5 --include \*.c | wc -l
101026
$ find ./ -name \*.c -exec cat '{}' \; | wc -l
11152601
```

This talk

An elegant small step semantics, and axiomatic semantics for goto, supporting:

- local variables (and pointers to those),
- mutual recursion,
- separation logic,
- soundness proof fully checked by Coq

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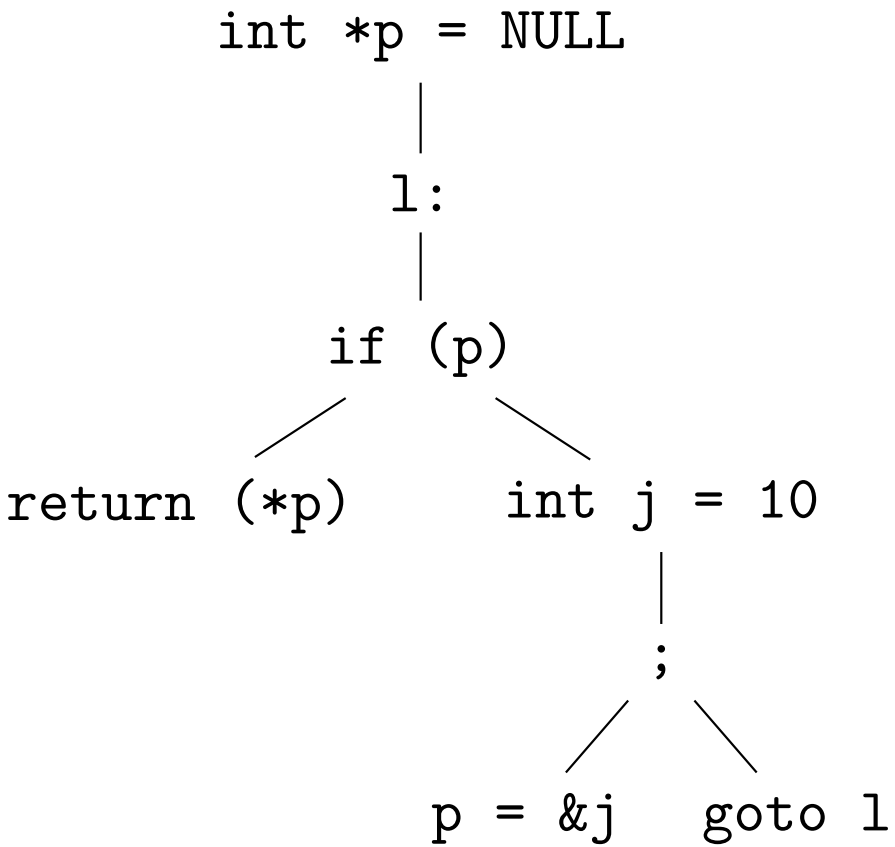
- Gotos and returns are also executed in small steps

Example



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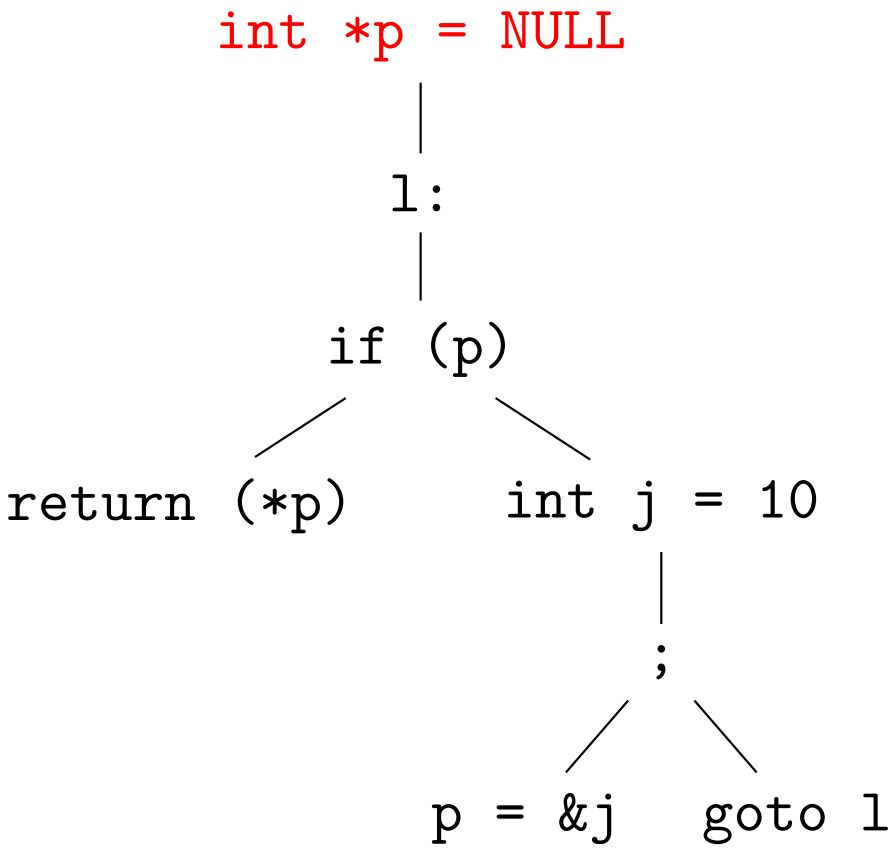

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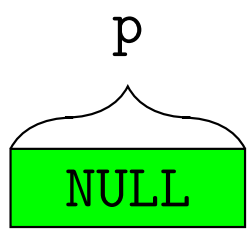

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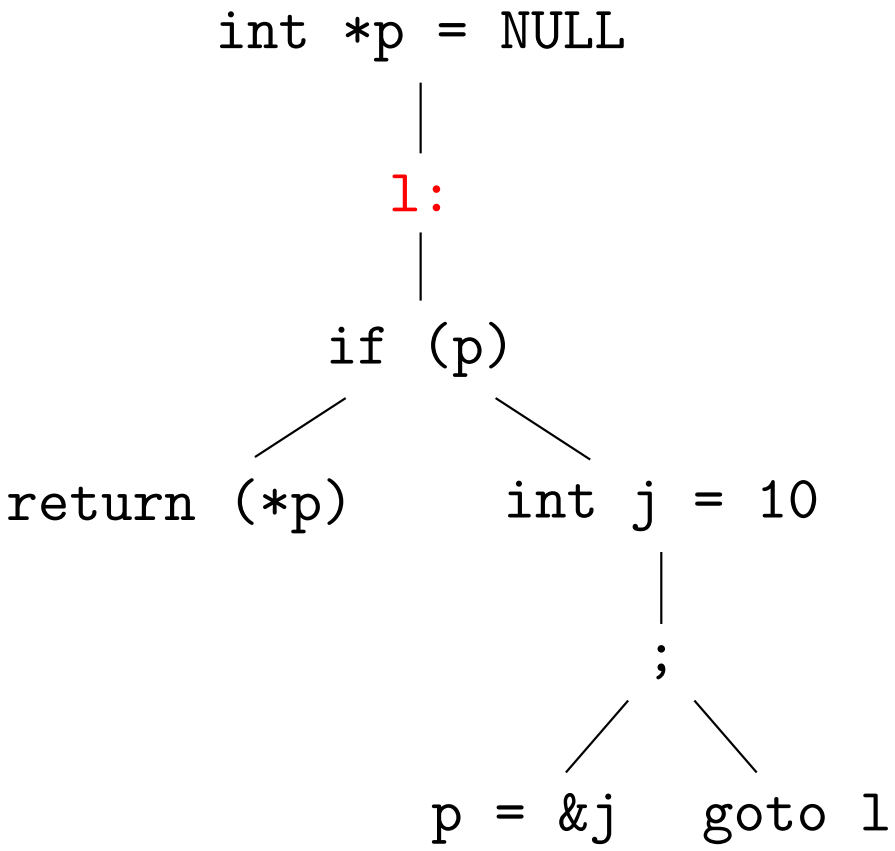

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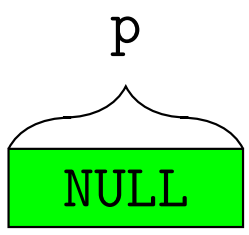

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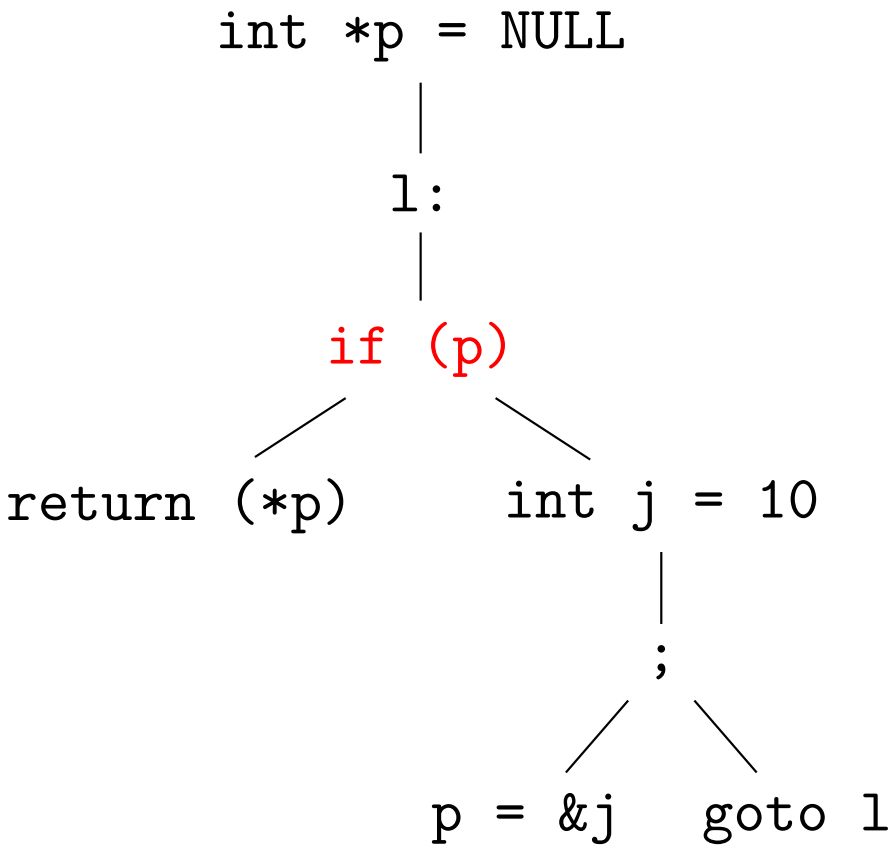

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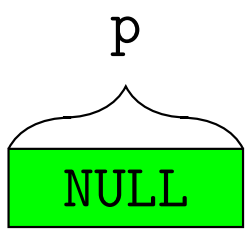

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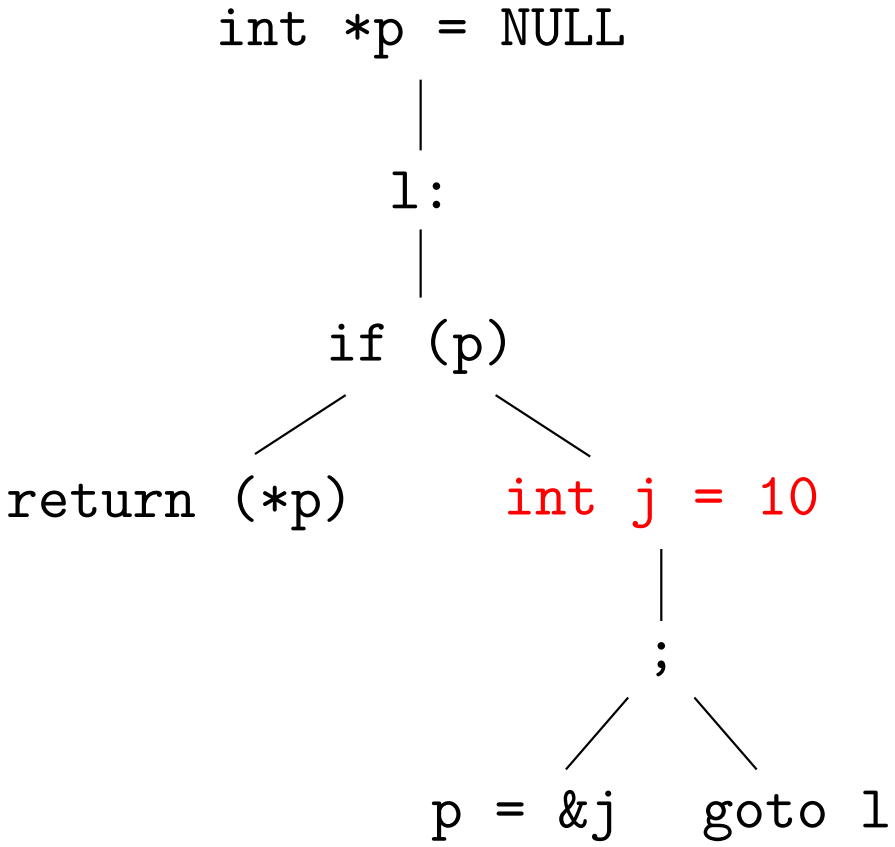

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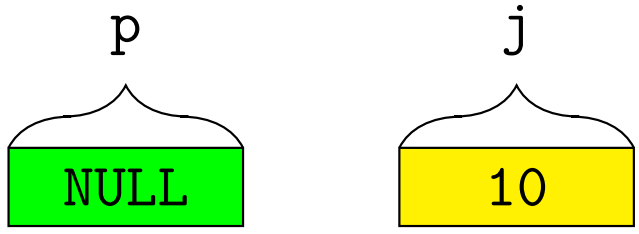

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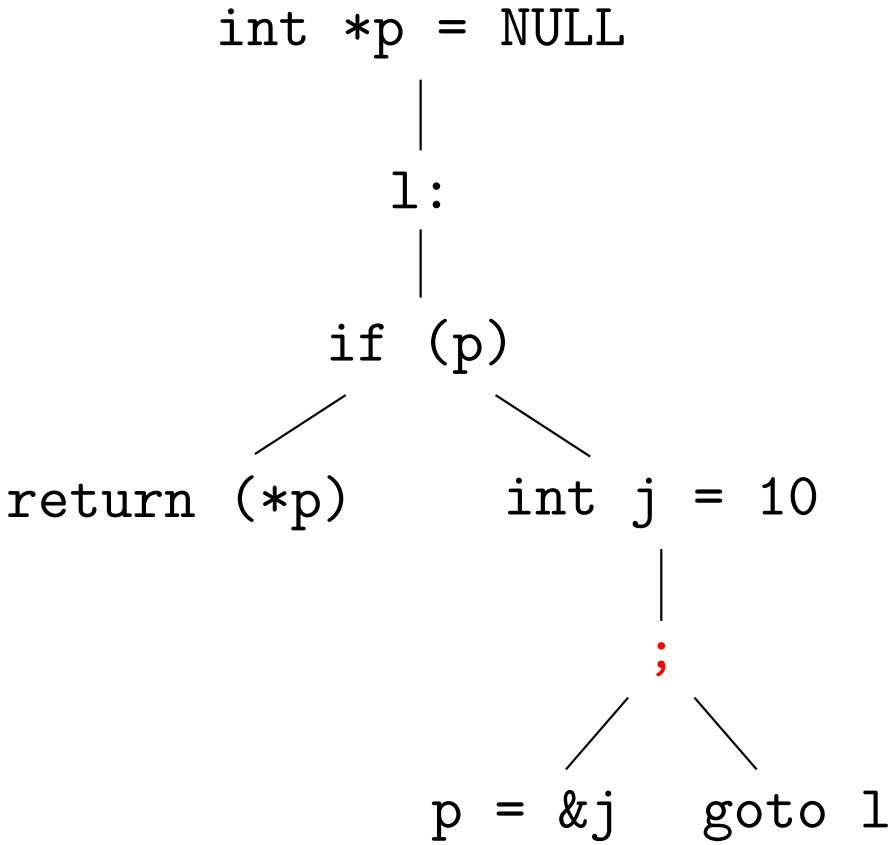

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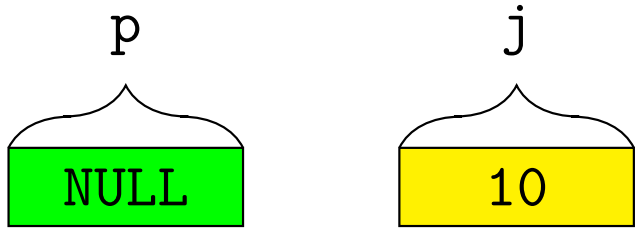

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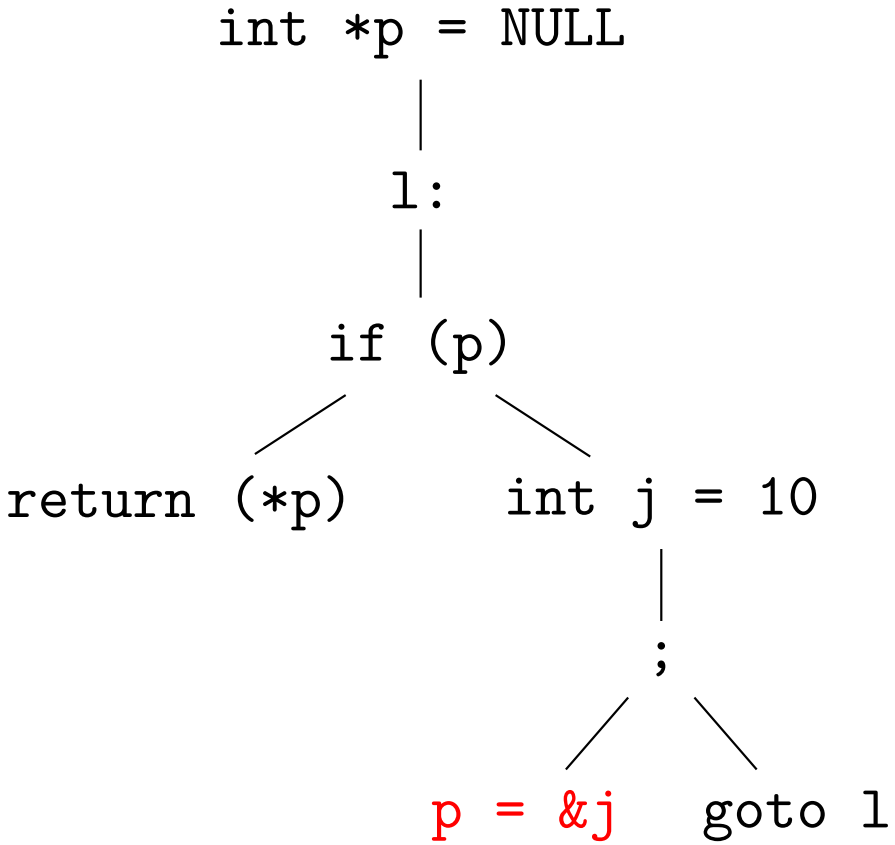

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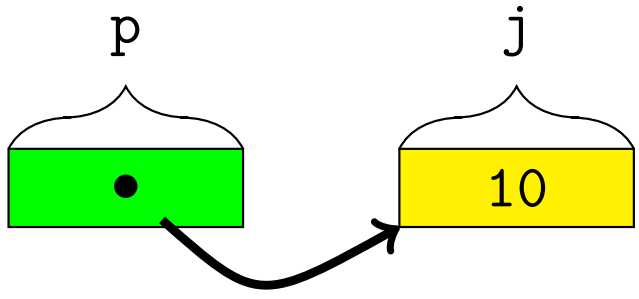

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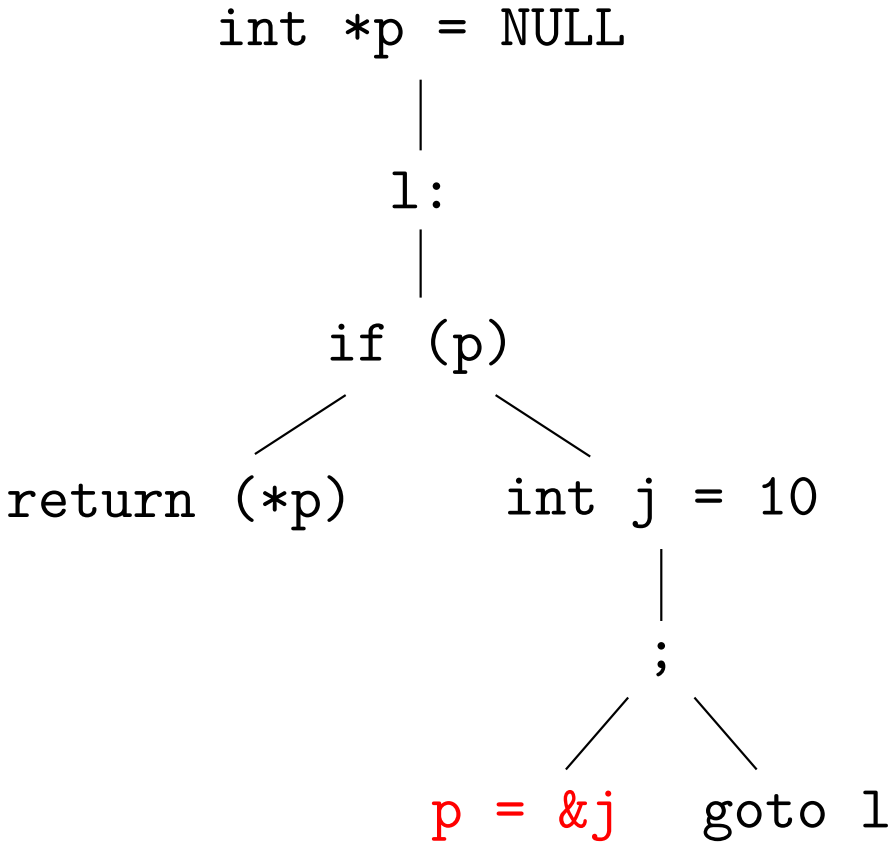

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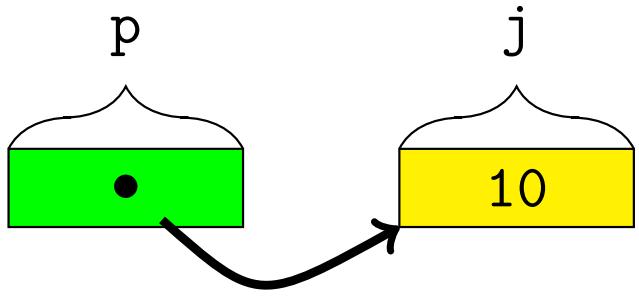

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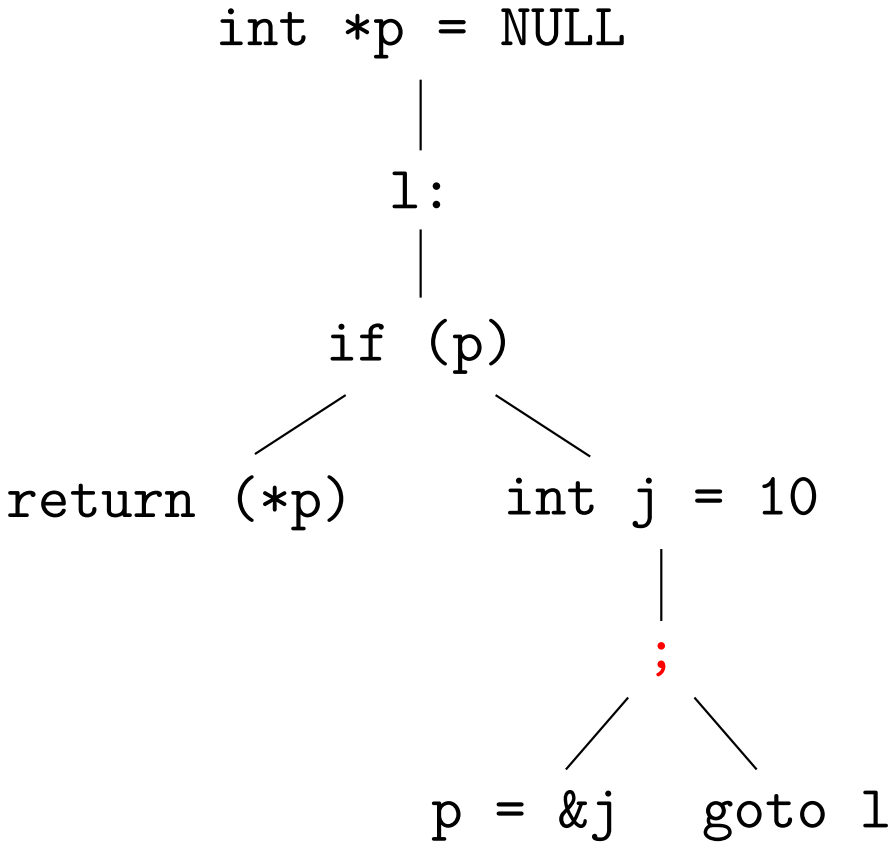

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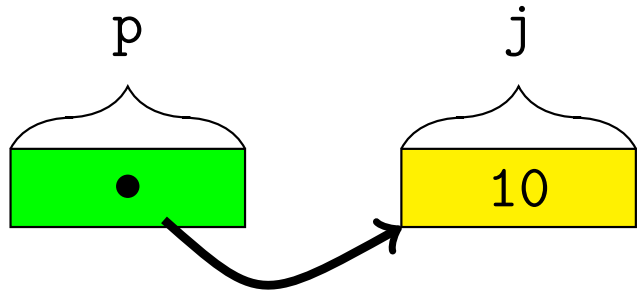

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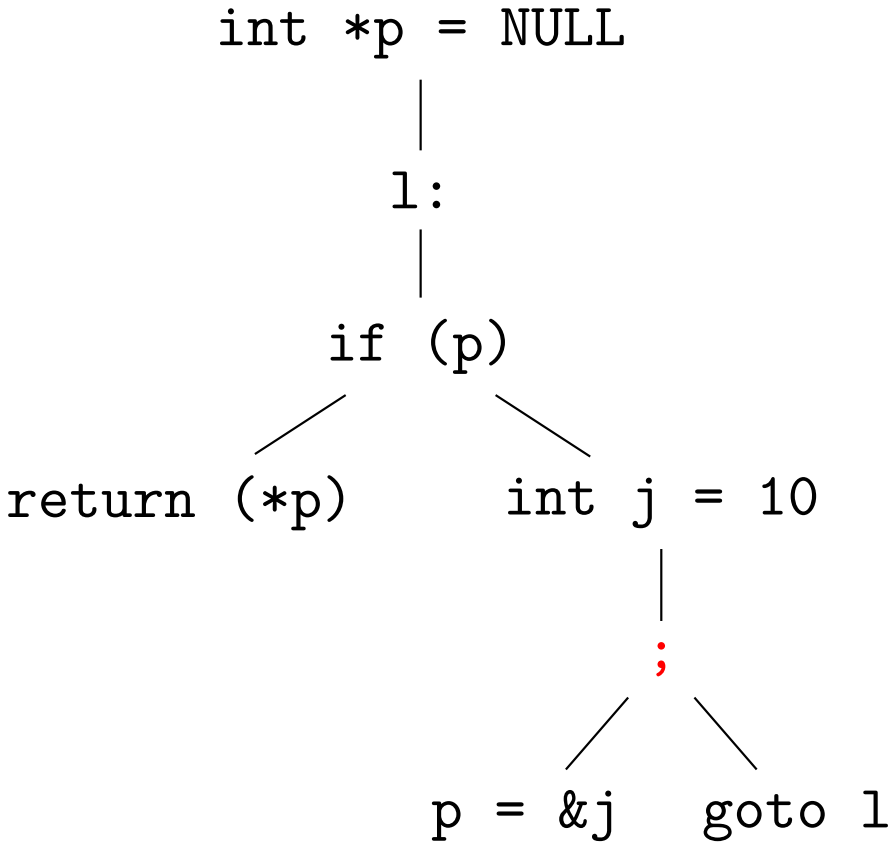

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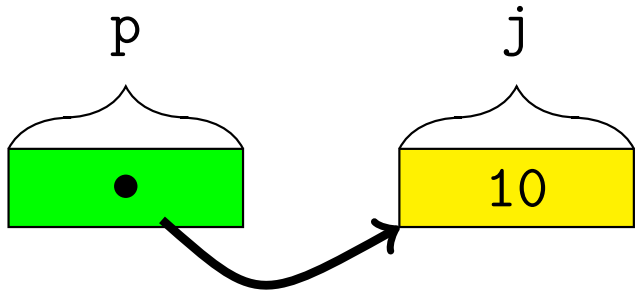

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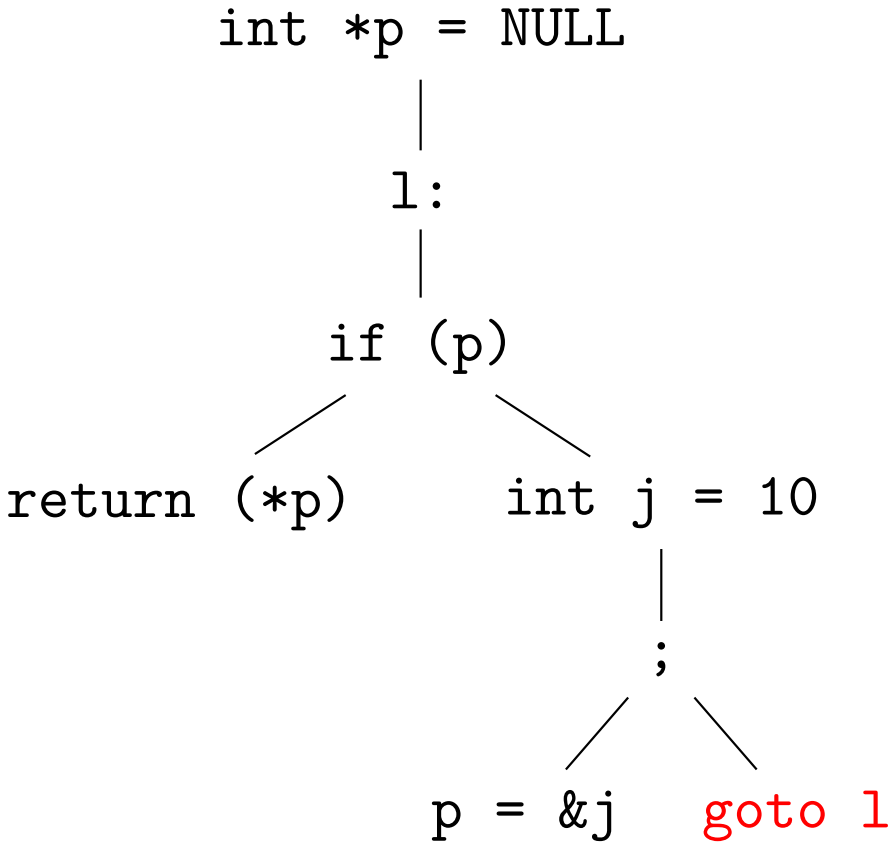

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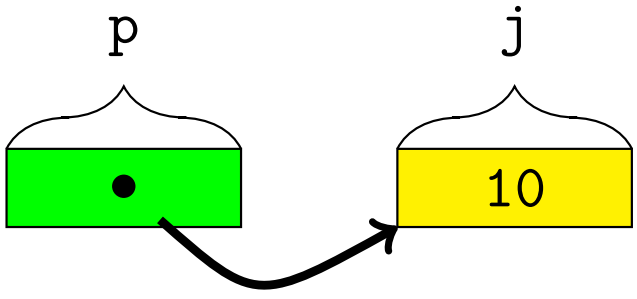

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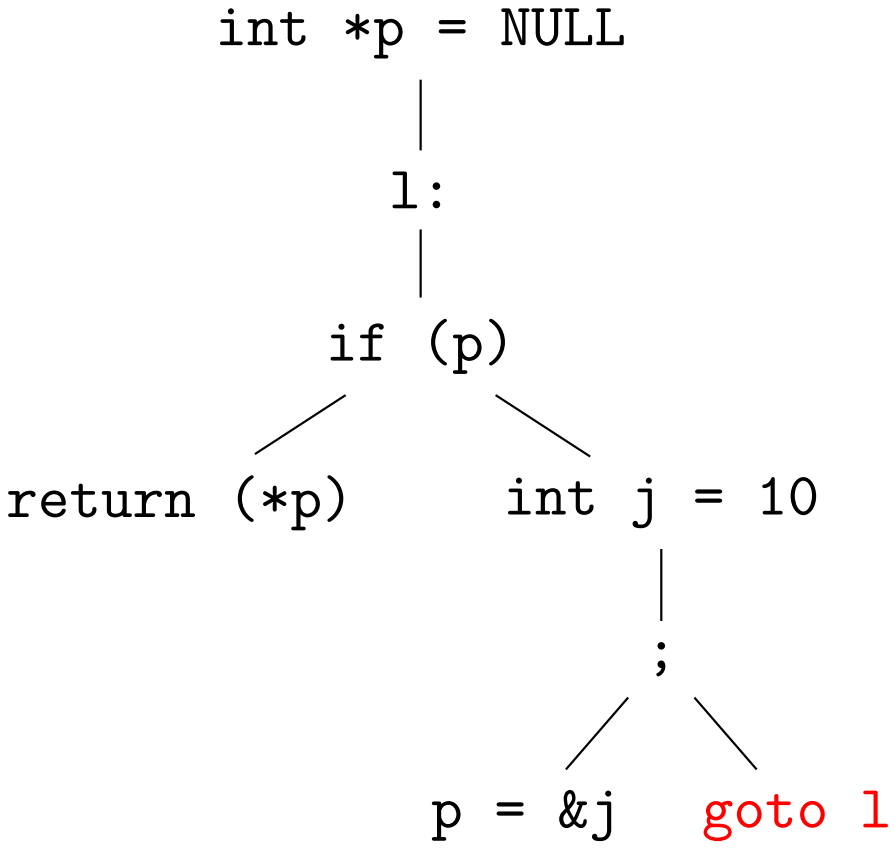

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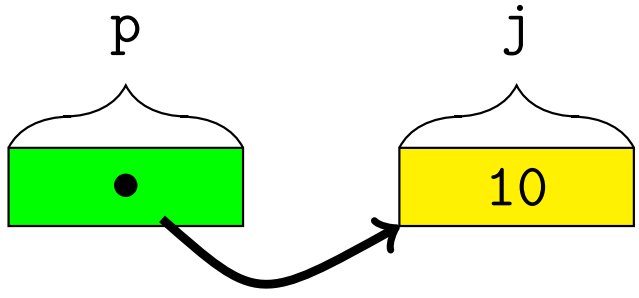

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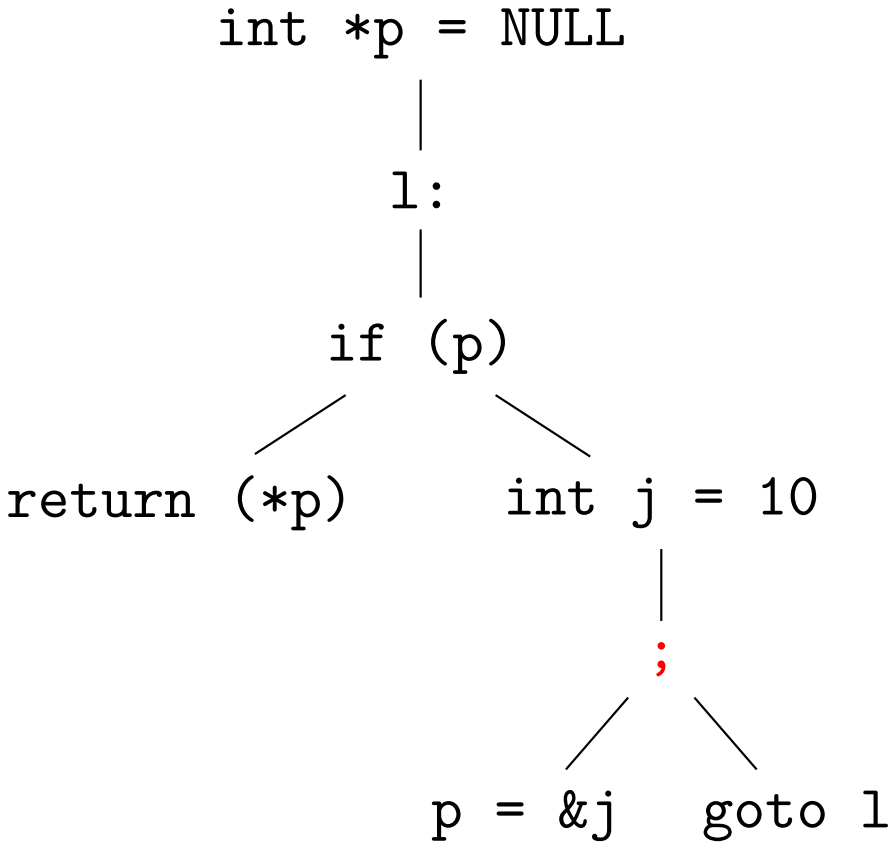

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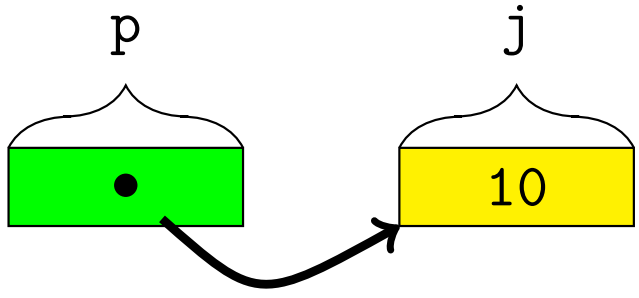

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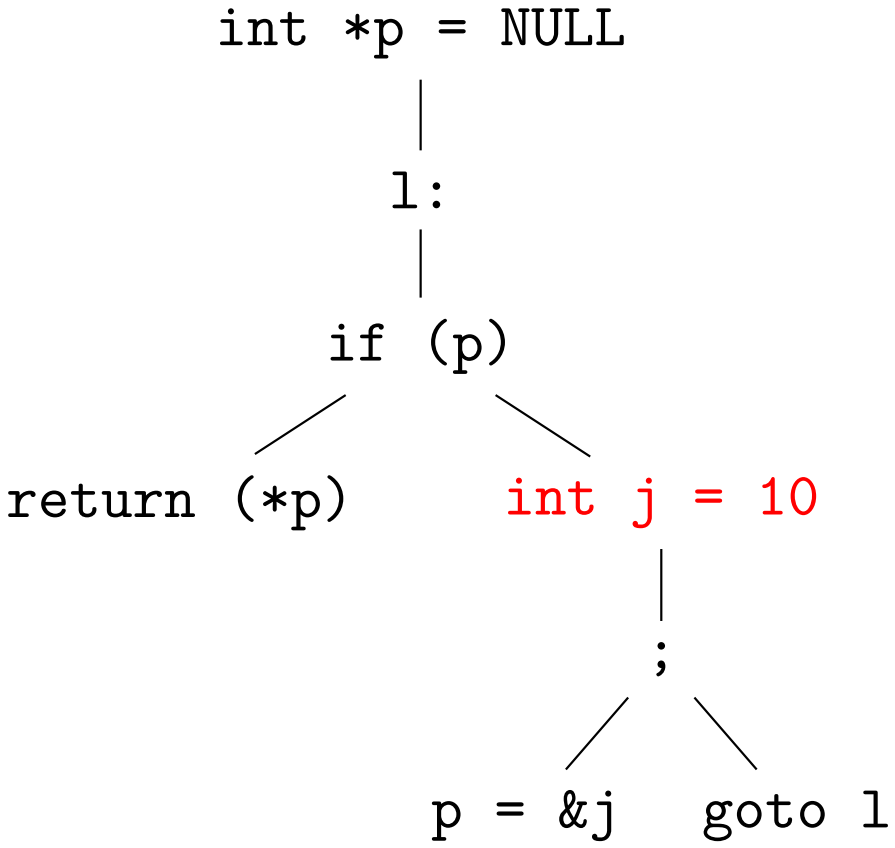

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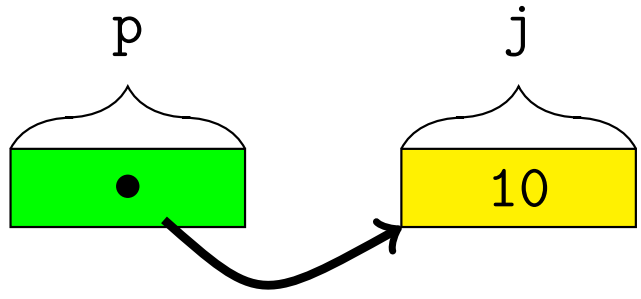

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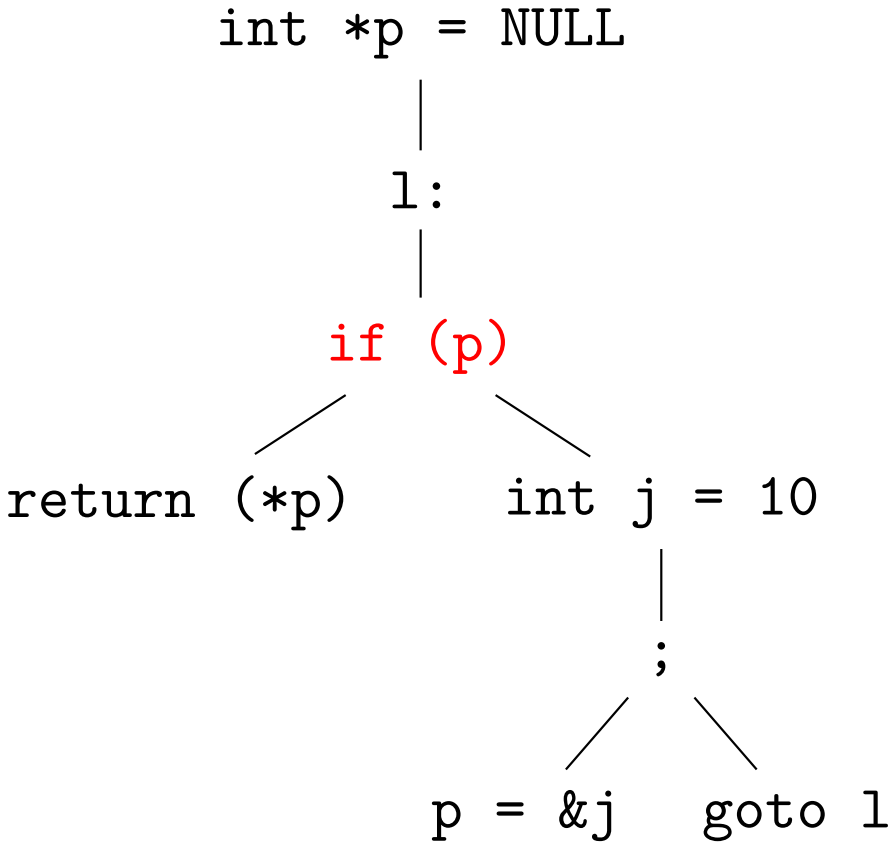

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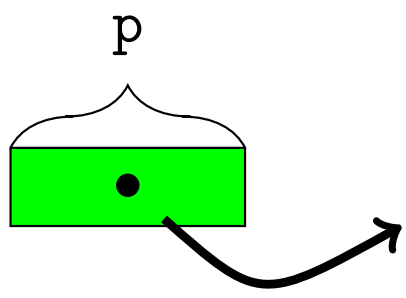

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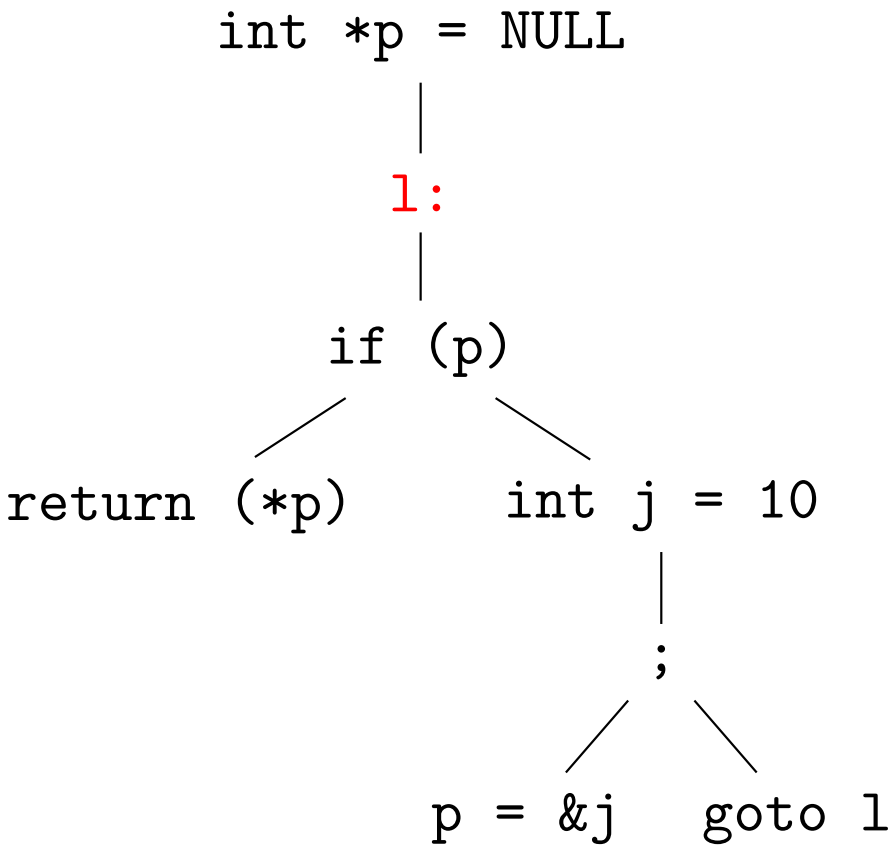

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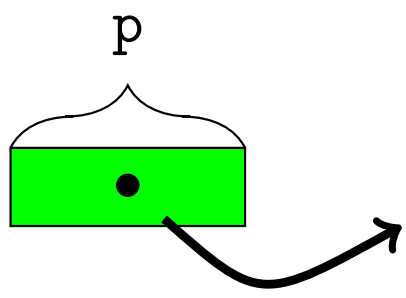

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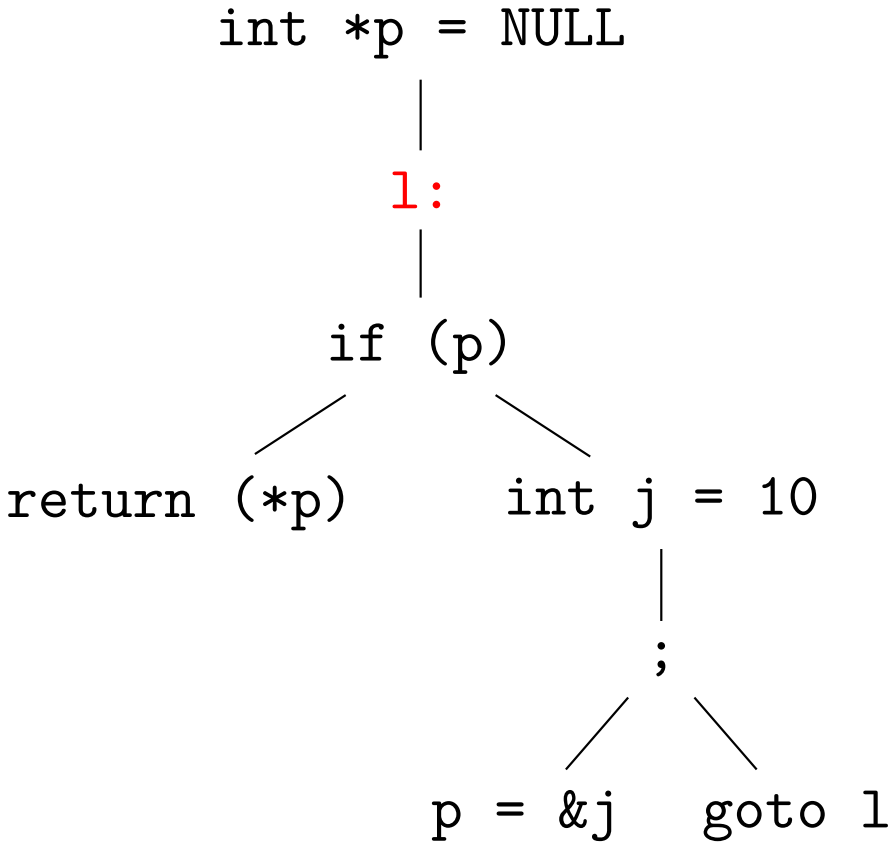

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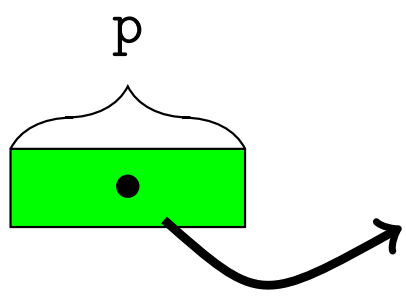

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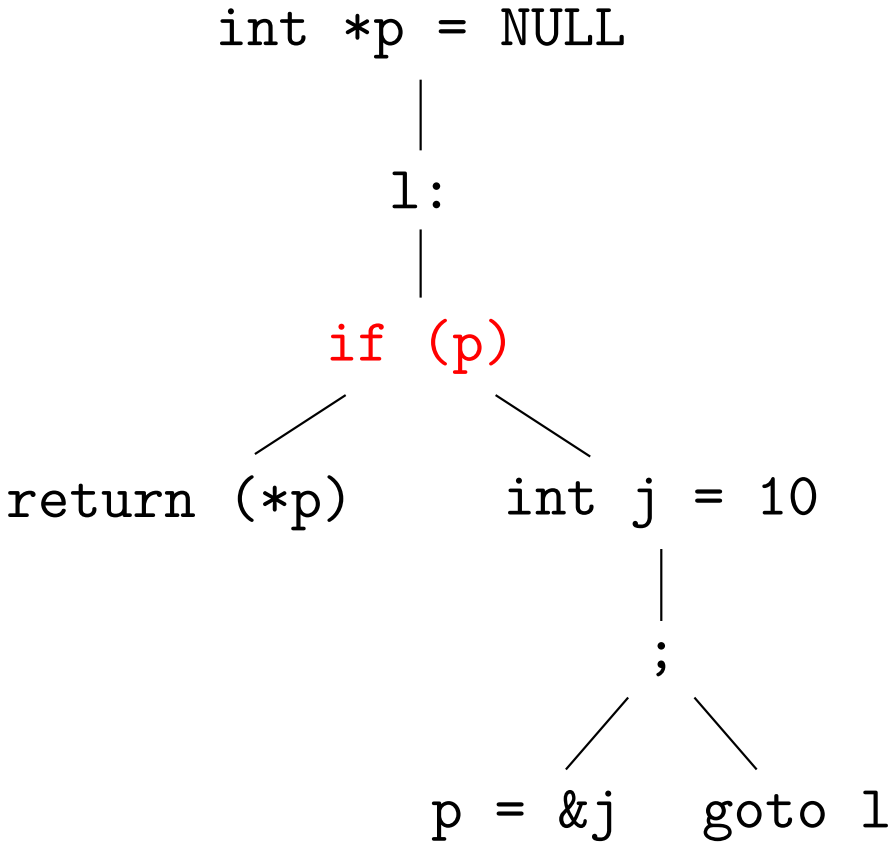

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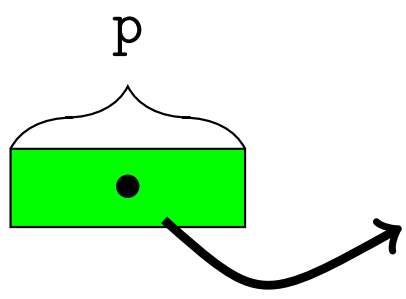

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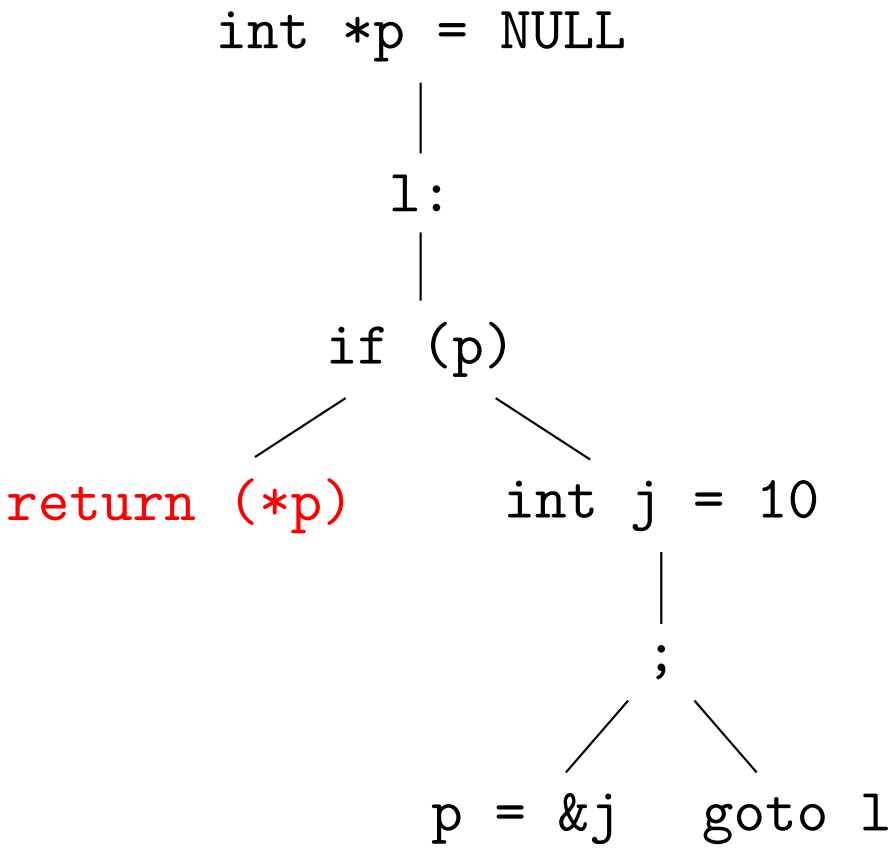

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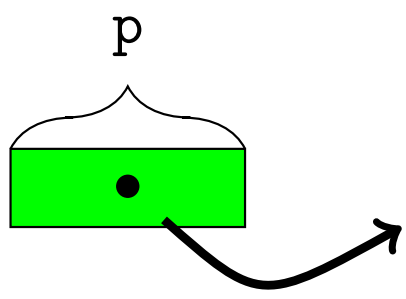
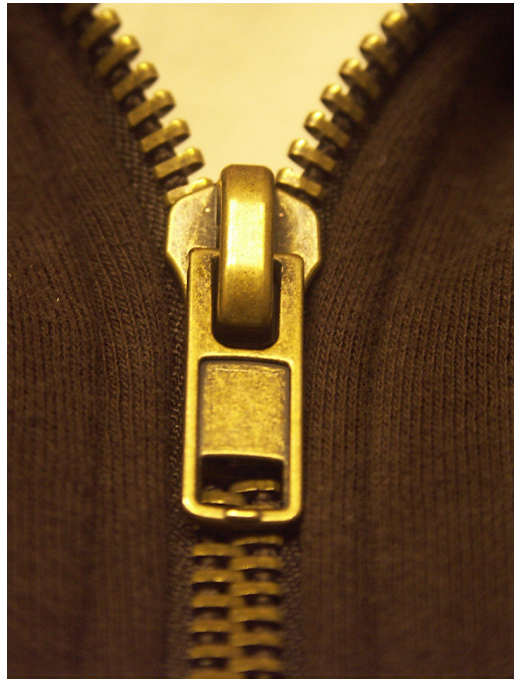


How to model the current *location* in the program

Huet's zipper



Purely functional way to store a pointer into a data structure

Statement contexts

- Statements:

$$s ::= \texttt{block}\ s \mid e_l := e_r \mid f(\vec{e}) \mid \texttt{skip} \mid \texttt{goto}\ l$$
$$\mid l : s \mid s_1 \,;\, s_2 \mid \texttt{if}\ (e)\ s_1\ s_2 \mid \texttt{return}$$

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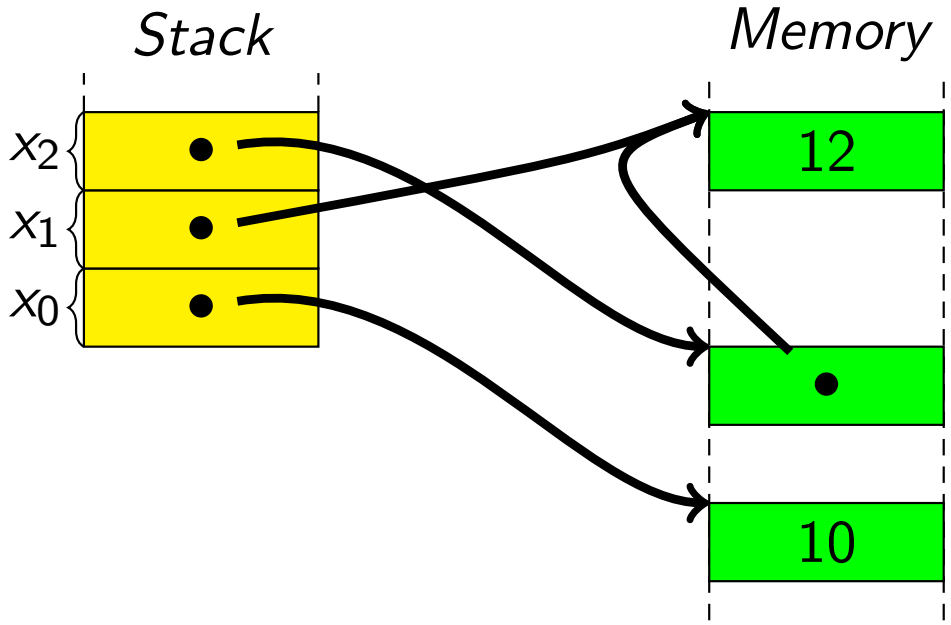
- A pair $(\vec{E_{\mathcal{S}}},\ s)$ forms a zipper for statements, where
  - $\vec{E_{\mathcal{S}}}$ is a statement turned inside-out
  - s is the focused substatement

Stacks

- A *stack* is a list of memory indexes
- A variable x_i refers to the ith element on the stack



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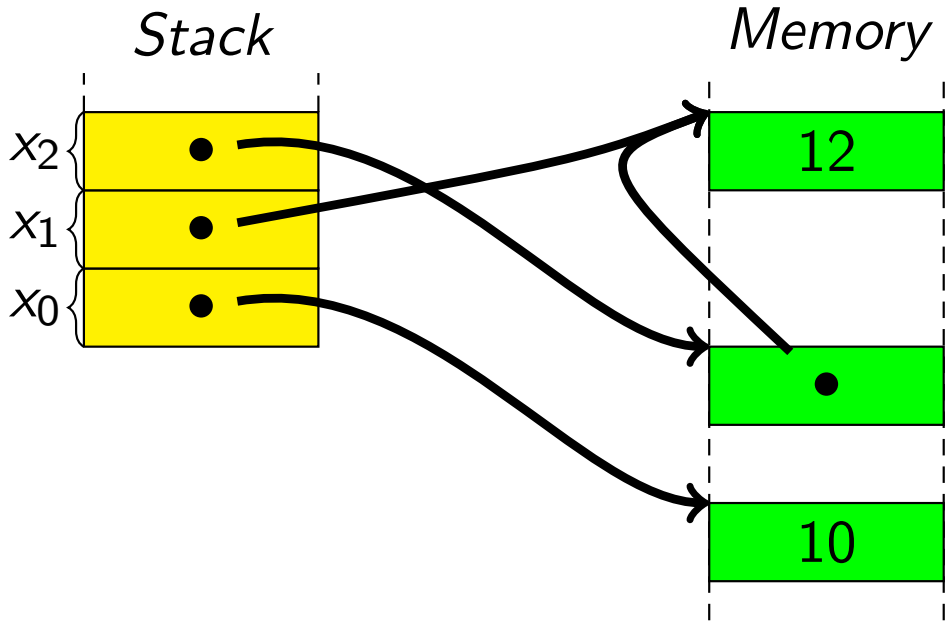

- Uniform way of dealing with pointers to local variables

Program contexts (1)

Singular program contexts:

$$E ::= E_{\mathcal{S}} \mid \texttt{block}_b\ \Box \mid \text{call } f\ \vec{e} \mid \text{params } \vec{b}$$

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Program contexts k are lists of singular program contexts

Program contexts (2)

- Program contexts contain the stack:

$$\begin{aligned}
\text{getstack}\ (E_{\mathcal{S}} :: k) &:= \text{getstack}\ k \\
\text{getstack}\ (\texttt{block}_b\ \square :: k) &:= b :: \text{getstack}\ k \\
\text{getstack}\ (\text{call}\ f\ \vec{e} :: k) &:= [\,] \\
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- Remark: not $\text{getstack}\ (\text{call}\ f\ \vec{e} :: k) := \text{getstack}\ k$

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A state $\mathbf{S}(k, \phi, m)$ consists of a program context k, *focus* ϕ, and memory m

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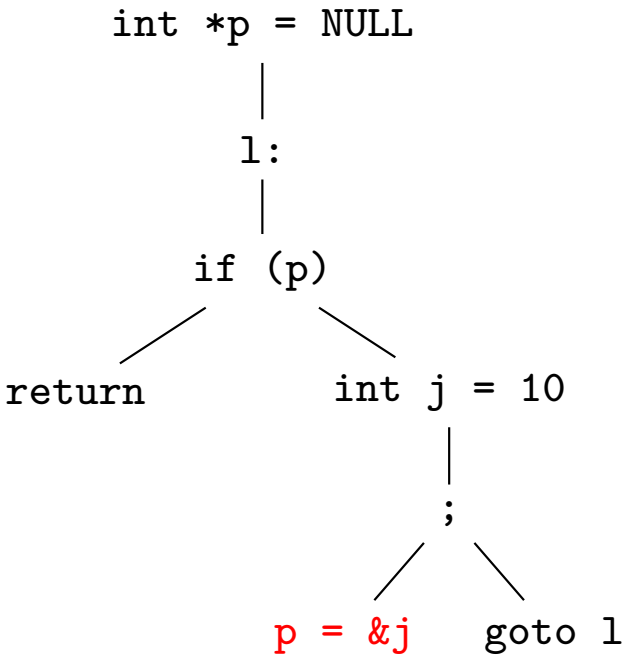
- $\overline{\text{return}}$ returning from a function

Example



The corresponding state is $\mathbf{S}(k, \phi, m)$, where:

- $k = [$
  $\square\,;\texttt{goto}\ l,$
  $x_0 := \texttt{int}\ 10\,;\square,$
  $\texttt{block}_{b_j}\ \square,$
  $\texttt{if}\ (\texttt{load}\ x_0)\ \texttt{return}\ \square,$
  $l : \square,$
  $x_0 := \texttt{NULL}\,;\square,$
  $\texttt{block}_{b_p}\ \square$
  $]$
- $\phi = (\nearrow,\ x_1 := x_0)$
- $m = \{b_p \mapsto \texttt{ptr}\ b_j, b_j \mapsto 10\}$

The small step semantics

Some rules:

- $\mathbf{S}(k, (\searrow, e_1 := e_2), m) \rightarrow \mathbf{S}(k, (\nearrow, e_1 := e_2), m[a := v])$ for a and v s.t. $[\![e_1]\!]_{k,m} = \texttt{ptr}\ a$, $[\![e_2]\!]_{k,m} = v$ and $m\, a \neq \bot$.

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- $\mathbf{S}(k, (\searrow, f(\vec{e})), m) \rightarrowtriangle \mathbf{S}(\mathrm{call}\ f\ \vec{e} :: k, \overline{\mathrm{call}}\ f\ \vec{v}, m)$ provided that $[\![e_i]\!]_{k,m} = v_i$ for each i

The small step semantics

Lemma

The small step semantics behaves as traversing through a zipper. That is, if

$$\mathbf{S}(k, (d, s), m) \rightarrowtriangle_k^* \mathbf{S}(k, (d', s'), m')$$

then $s = s'$.

Hoare triples

Traditional *Hoare triples* are of the shape

$$\{P\}\, s\, \{Q\}$$

Intuitive meaning:

- If P holds for the state before execution of s,
- and execution of s terminates,
- then Q will hold afterwards

Extended Hoare 'triples' (1)

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- R has to hold to execute a `return`

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Observations:

- The assertions P, Q, J and R correspond to the four directions $\searrow$, $\nearrow$, $\curvearrowright$ and $\Uparrow$

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Observations:

- The assertions P, Q, J and R correspond to the four directions $\searrow$, $\nearrow$, $\curvearrowright$ and $\Uparrow$
- We thus treat the sextuple as

$$\Delta;\ \bar{P} \vdash s$$

where $\bar{P}\searrow = P$, $\bar{P}\nearrow = Q$, $\bar{P}\,(\curvearrowright I) = J\,I$ and $\bar{P}\Uparrow = R$

Some Hoare rules

Weakening:

$$\frac{\begin{array}{c}(\forall l \in \mathsf{labels}\ s\,.\ J'\,l \to J\,l) \quad (\forall l \notin \mathsf{labels}\ s\,.\ J\,l \to J'\,l) \quad R \to R' \\ P' \to P \quad \Delta;\ J;\ R \vdash \{P\}\,s\,\{Q\} \quad Q \to Q'\end{array}}{\Delta;\ J';\ R' \vdash \{P'\}\,s\,\{Q'\}}$$

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Used for local reasoning

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Using our alternative notation:

$$\frac{\Delta;\ \bar{P} \vdash s}{\Delta;\ \bar{P} * A \vdash s}$$

The block scope variable rule

The assertion $A\uparrow$ lifts the DeBruijn indexes in A

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Using our alternative notation

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Example

```
void swap(int *p, int *q) {
  int z = *p; *p = *q; *q = z;
}
```

Example

$$\{x_0 \mapsto p * x_1 \mapsto q * p \mapsto y * q \mapsto z\}$$

$$\texttt{block}\,(\quad x_0 := \texttt{load}\,(\texttt{load}\,x_1)\,;\quad \texttt{load}\,x_1 := \texttt{load}\,(\texttt{load}\,x_2)\,;\quad \texttt{load}\,x_2 := \texttt{load}\,x_0\quad)$$

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- Prove $\Delta;\ J;\ R \vdash \{P\}\, s\, \{Q\}$ implies $\Delta;\ J;\ R \vDash \{P\}\, s\, \{Q\}$
- Tricky definition of $\Delta;\ J;\ R \vDash \{P\}\, s\, \{Q\}$ because
  - The frame rule
  - Undefined behavior
  - Non-local control
  - Mutual recursion

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- 3500 lines of code

Future research

- Expressions with side effects
- The C type system
- Non-aliasing restrictions
- Verification condition generator in Coq
- Correspondence with CompCert

Questions

Sources: see `http://robbertkrebbers.nl/research/ch2o/`