Separation Logic for Non-local Control Flow

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Joint work with Freek Wiedijk

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What is this program supposed to do?

```
int *p = NULL;
l: if (p) {
  return (*p);
} else {
  int j = 10;
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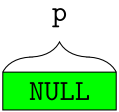

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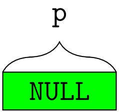

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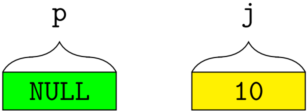

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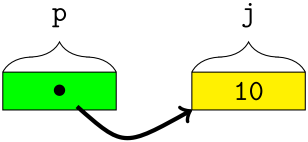

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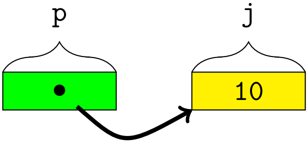

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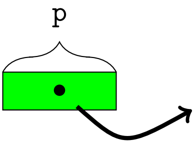

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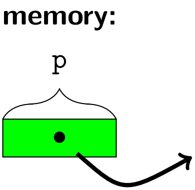

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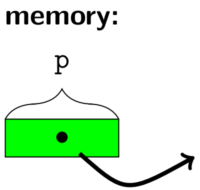

It exhibits undefined behavior, thus it may do *anything*

Why is catching undefined behavior important

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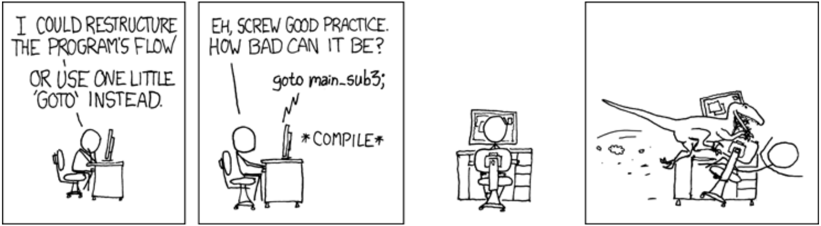
- Not catching it means that
  - programs can be proven to be correct with respect to the formal semantics ...
  - whereas they may crash when compiled with an actual compiler

Goto considered harmful?



http://xkcd.com/292/

Goto considered harmful?

Not necessarily:

$$\vdash \{P\}\ldots\ \texttt{goto main\_sub3;}\ \ldots\{Q\}$$

Why goto

Goto can be useful

- Breaking from multiple nested loops

```
for (int i = 0; i++; i < n) {
  // do something here

  for (int j = i; j++; j < m) {
    // do some work here

    goto outer;
  }
}
outer:;
```

Why goto

Goto can be useful

- Breaking from multiple nested loops
- Systematically cleaning up resources after errors

```
if (!openDataFile())
  goto quit;
if (!getDataFromFile())
  goto closeFileAndQuit;
if (!allocateSomeResources)
  goto freeResourcesAndQuit;

// do actual work here

freeResourcesAndQuit: // free resources
closeFileAndQuit: // close file
quit: // quit!
```

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Goto is used in practice

- The Linux kernel contains about $\sim$ 100.000 uses of goto

```
$ grep -w -r goto ~/src/linux-3.5.5 --include \*.c | wc -l
101026
```

This talk

An elegant small step semantics, and axiomatic semantics for goto, supporting:

- local variables (and pointers to those),
- mutual recursion,
- separation logic,
- soundness proof fully checked by Coq

Approach

- Small step semantics by traversal through the program

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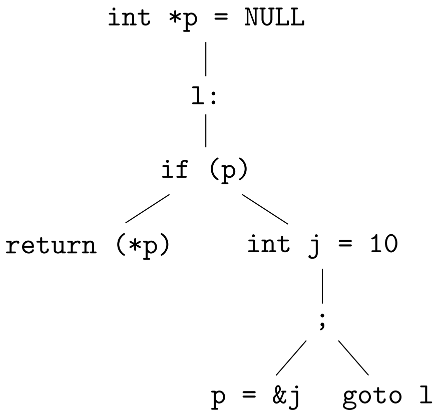
  - $\curvearrowright$`l` to a label `l:` after a `goto l`
  - $\upuparrows$ to the top after a `return`
- Gotos and returns are also executed in small steps

Example



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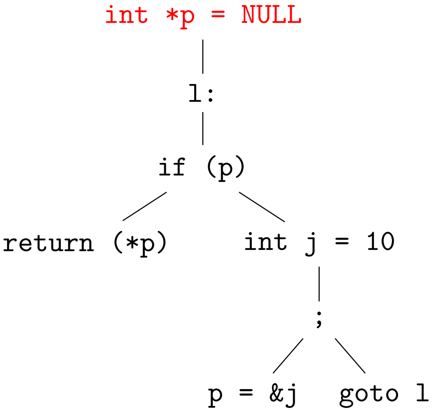
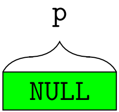


direction:

memory:



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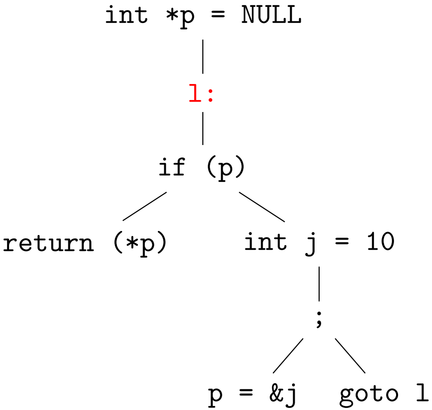

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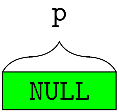

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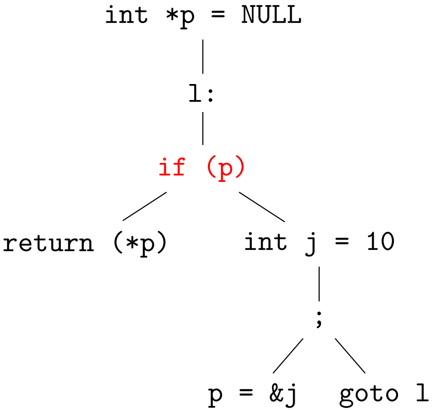

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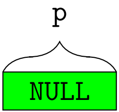

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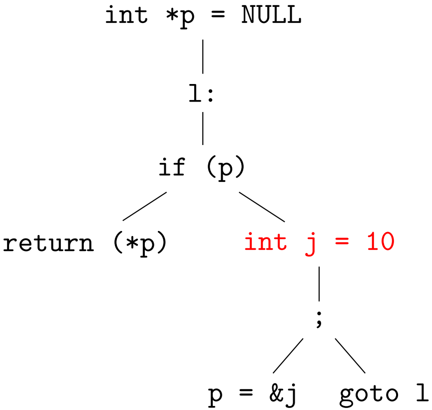
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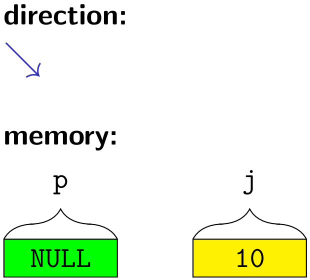
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memory:
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10

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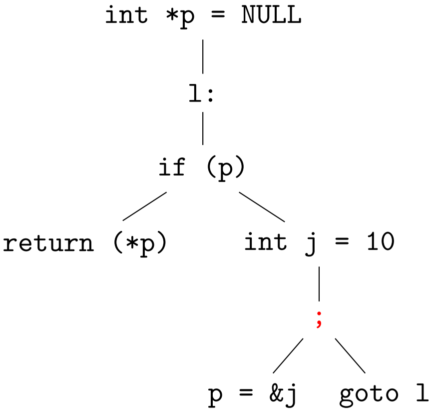

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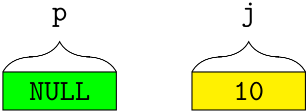

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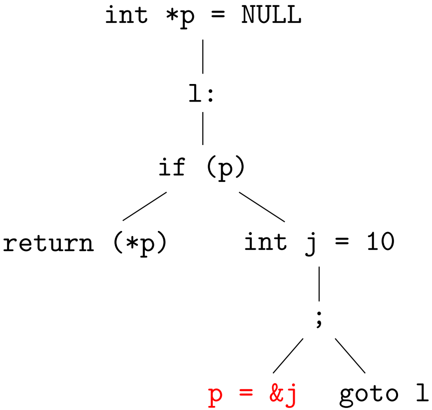

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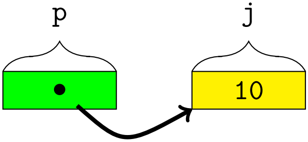

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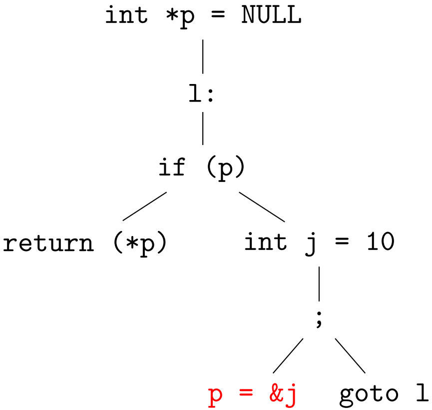

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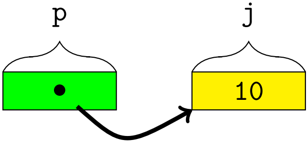

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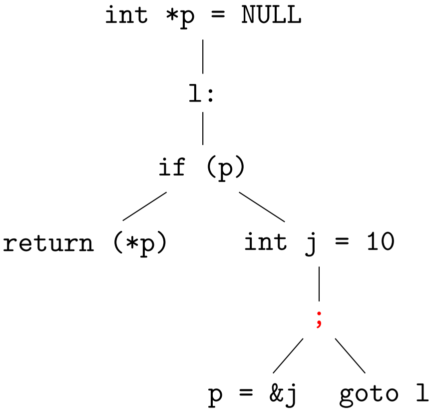

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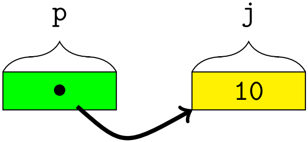

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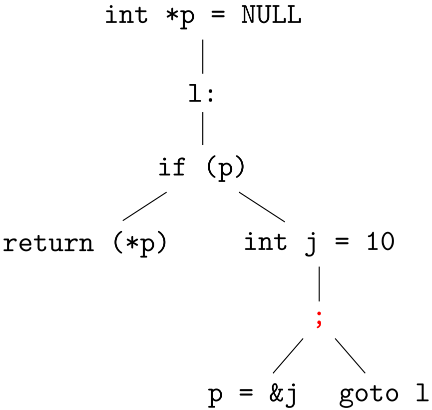

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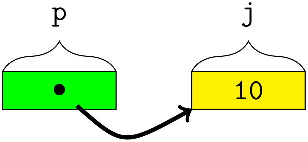

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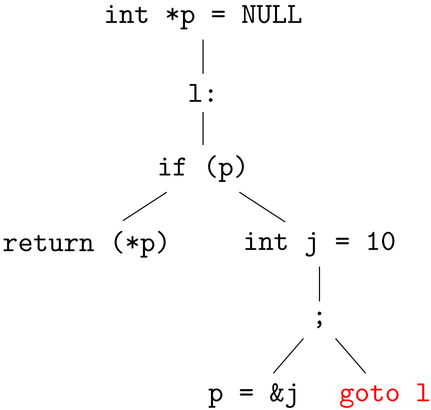

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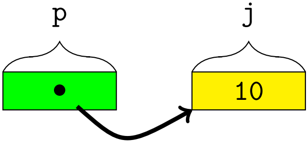

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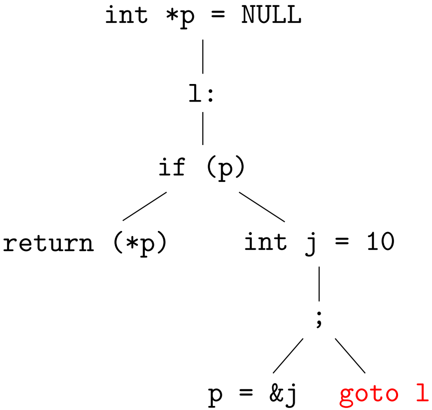

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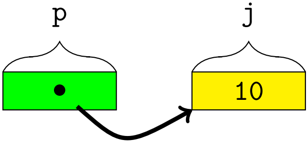

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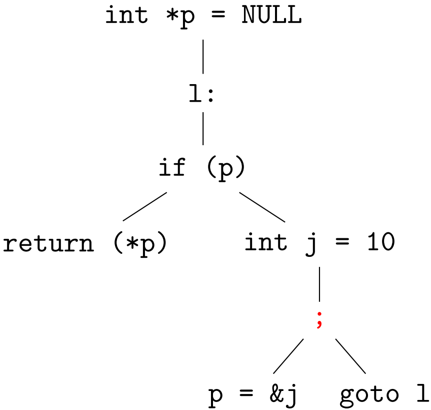

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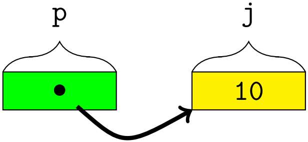

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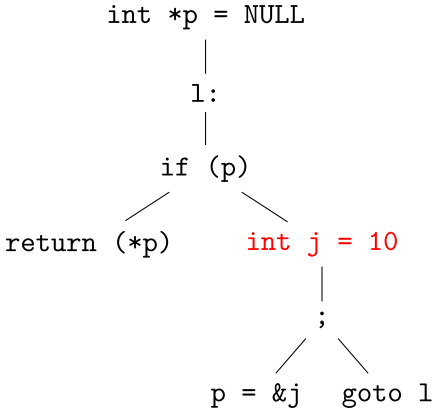

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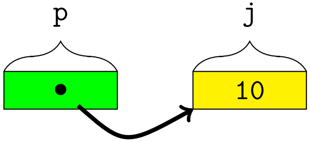

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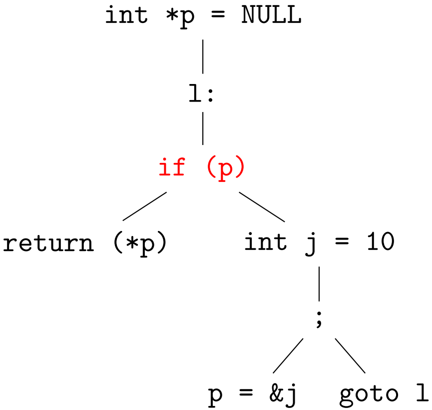

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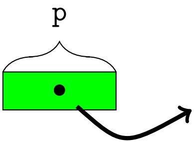

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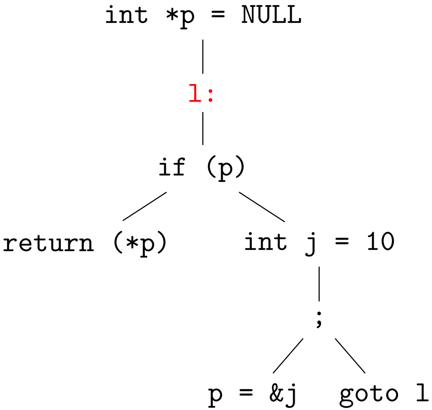

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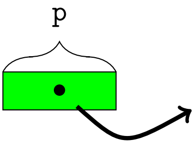

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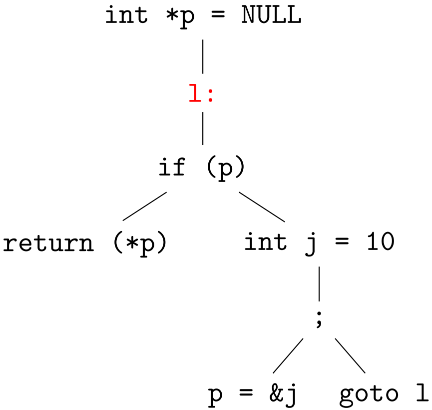

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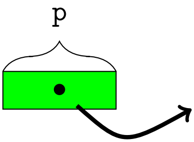

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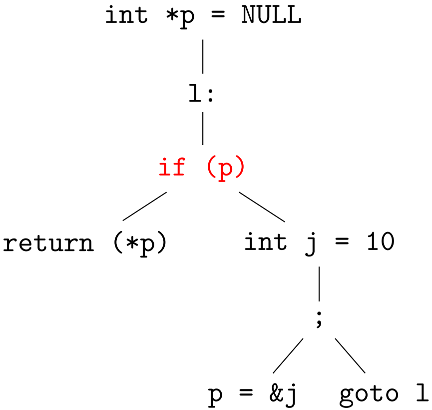

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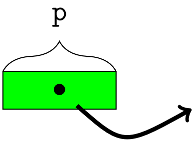

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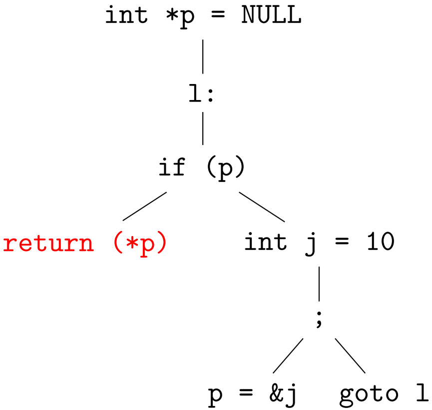

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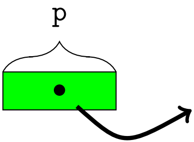
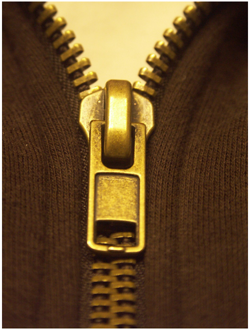


How to model the current *location* in the program

Huet's zipper



Purely functional way to store a pointer into a data structure

Program contexts (1)

- Statements:

$$\begin{aligned} s ::= {} & \texttt{block}\ s \mid e_l := e_r \mid f(\vec{e}) \mid \texttt{skip} \mid \texttt{goto}\ l \mid \\ & s_1\,;s_2 \mid \texttt{if}\ (e)\ s_1\ s_2 \mid \texttt{return} \end{aligned}$$

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$$E_{\mathcal{S}} ::= \square \,;\, s_2 \mid s_1 \,;\, \square \mid \texttt{if}\ (e)\ \square\ s_2 \mid \texttt{if}\ (e)\ s_1\ \square \mid l : \square$$

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- A pair $(\vec{E_{\mathcal{S}}}, s)$ forms a zipper for statements, where
  - $\vec{E_{\mathcal{S}}}$ is a statement turned inside-out
  - s is the focused substatement

Program contexts (2)

Singular program contexts:

$$E ::= E_{\mathcal{S}} \mid \texttt{block}_b \, \square \mid \text{call } f \ \vec{e} \mid \text{params } \vec{b}$$

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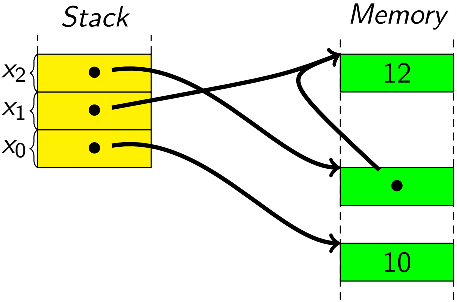
Program contexts k are lists of singular program contexts

Stacks (1)

- A *stack* is a list of memory indexes
- A variable x_i refers to the ith element on the stack



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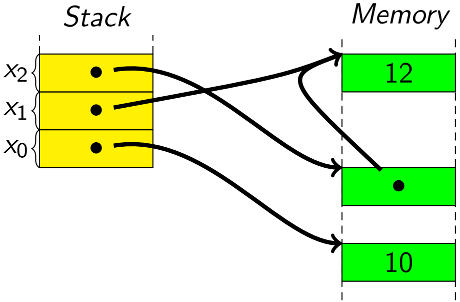

- Uniform way of dealing with pointers to local variables

Stacks (2)

- Program contexts contain the stack:

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\text{getstack}\ (E_{\mathcal{S}} :: k) &:= \text{getstack}\ k \\
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- Remark: not $\mathrm{getstack}\ (\mathrm{call}\ f\ \vec{e} :: k) := \mathrm{getstack}\ k$

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A state $\mathbf{S}(k, \phi, m)$ consists of a program context k, *focus* ϕ, and memory m

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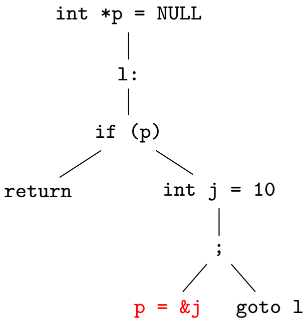
- $\overline{\mathsf{return}}$ returning from a function

Example



The corresponding state is $\mathbf{S}(k,\, \phi,\, m)$, where:

- $k = [$
  $\square\,;\mathtt{goto}\ l,$
  $x_0 := \mathtt{int}\ 10\,;\square,$
  $\mathtt{block}_{b_j}\ \square,$
  $\mathtt{if}\ (\mathtt{load}\ x_0)\ \mathtt{return}\ \square,$
  $l : \square,$
  $x_0 := \mathtt{NULL}\,;\square,$
  $\mathtt{block}_{b_p}\ \square$
  $]$
- $\phi = (\nearrow,\, x_1 := x_0)$
- $m = \{b_p \mapsto \mathtt{ptr}\ b_j,\, b_j \mapsto 10\}$

The small step semantics

Some rules:

- $\mathbf{S}(k, (\searrow, e_l := e_r), m) \rightharpoondown \mathbf{S}(k, (\nearrow, e_l := e_r), m[a := v])$ provided that $[\![\, e_1 \,]\!]_{k,m} = \mathtt{ptr}\ a$, $[\![\, e_2 \,]\!]_{k,m} = v$, and $m\,a \neq \bot$

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  provided that $[\![\ e_i\ ]\!]_{k,m} = v_i$ for each i

The small step semantics

Lemma

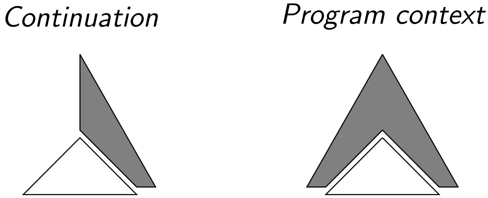
The small step semantics behaves as traversing through a zipper. That is, if

$$\mathbf{S}(k,\ (d,\ s),\ m) \rightharpoonup^*_k \mathbf{S}(k,\ (d',\ s'),\ m')$$

then $s = s'$.

Program contexts versus continuations

- Program contexts also contain the part of the program that has already been executed



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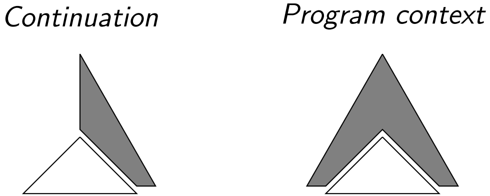

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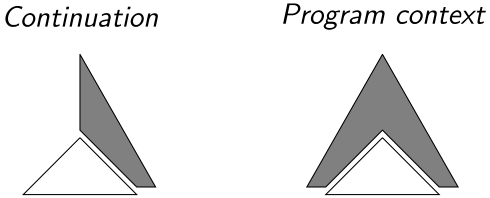

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- Program contexts implicitly contain the stack

Hoare triples

Traditional *Hoare triples* are of the shape

$$\{P\}\, s\, \{Q\}$$

Intuitive meaning:

- If P holds for the state before execution of s,
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We use a *shallow embedding*, *i.e. assertions* P, Q are predicates on the stack and memory

Extended Hoare 'triples' (1)

Our *Hoare sextuples* are of the shape

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Observations:

- The assertions P, Q, J and R correspond to the four directions $\searrow$, $\nearrow$, $\curvearrowright$ and $\upuparrows$

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- We thus treat the sextuple as

$$\Delta;\, \bar{P} \vdash s$$

where $\bar{P}\searrow\, = P$, $\bar{P}\nearrow\, = Q$, $\bar{P}\,(\curvearrowright l) = J\, l$ and $\bar{P}\upuparrows\, = R$

Some Hoare rules

Weakening:

$$\frac{P \rightarrow P' \qquad \Delta;\ J;\ R \vdash \{P'\}\, s\, \{Q'\} \qquad Q' \rightarrow Q}{\Delta;\ J;\ R \vdash \{P\}\, s\, \{Q\}}$$

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The frame rule

Used for local reasoning

$$\frac{\Delta;\ J;\ R \vdash \{P\}\, s\, \{Q\}}{\Delta;\ J * A;\ R * A \vdash \{P * A\}\, s\, \{Q * A\}}$$

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Using our alternative notation:

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The block scope variable rule

The assertion $A\!\uparrow$ lifts the DeBruijn indexes in A

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- Prove $\Delta;\, J;\, R \vdash \{P\}\, s\, \{Q\}$ implies $\Delta;\, J;\, R \vDash \{P\}\, s\, \{Q\}$
- Tricky definition of $\Delta;\, J;\, R \vDash \{P\}\, s\, \{Q\}$ because
  - The frame rule
  - Undefined behavior
  - Non-local control
  - Mutual recursion

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- 6500 lines, of which half on the actual formalization

Future research

- Expressions with side effects
- The C type system
- Non-aliasing restrictions
- Verification condition generator in Coq
- Correspondence with CompCert

Questions

Sources: see `http://robbertkrebbers.nl/research/ch2o/`