Aliasing restrictions of C11 formalized in Coq

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  int x = *p; *q = 314; return x;
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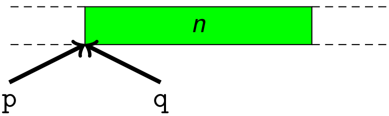

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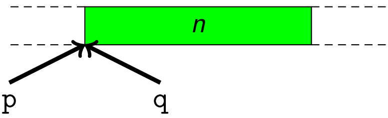

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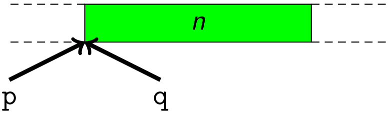

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Alias analysis: to determine whether pointers can alias

Aliasing with different types

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union { int x; float y; } u;
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return h(&u.x, &u.y);
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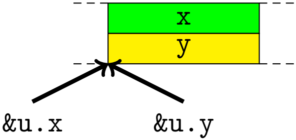

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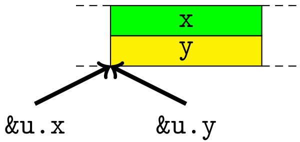

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C99/C11 allows **type-based alias analysis**:

- A compiler can assume that p and q do not alias
- Reads/writes with "the wrong type" yield undefined behavior

Undefined behavior in C

"Garbage in, garbage out" principle

- Programs with undefined behavior are not statically excluded
- Instead, these may do literally anything when executed
- Compilers are allowed to assume no undefined behavior occurs
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A formal C semantics should account for undefined behavior

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Interplay between low- and high-level

- Each object *should be* represented as a sequence of bits
  . . . which can be inspected and manipulated *in C*
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The standard is unclear on many of such difficulties
Opportunities for a formal semantics to resolve this unclarity!

Contribution

An abstract formal memory for C supporting

- Types (arrays, structs, unions, . . .)
- Strict aliasing restrictions (effective types)
- Byte-level operations
- Type-punning
- Indeterminate memory
- Pointers "one past the last element"
- Parametrized by an interface for integer types
- Formalized together with essential properties in Coq

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Others (e.g. CompCert)	Our approach
Memory: a finite map of cells which consist of arrays of bytes	
Pointers: pairs (x, i) where x identifies the cell, and i the offset into that cell	
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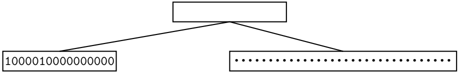
Three kinds of values

Our formal description has three kinds of values. For

```
struct { short x, *p; } s = { 33; &s.x }
```

we have:

- *A memory value* with arrays of bits as leafs:



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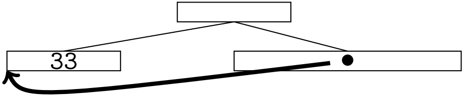

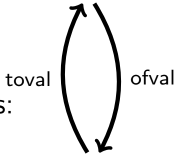

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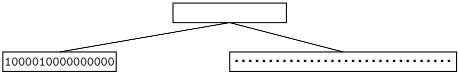

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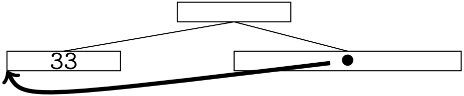

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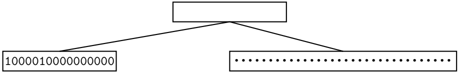

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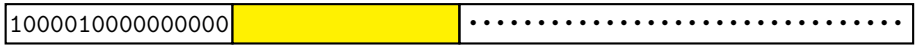

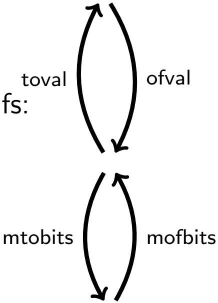

Bits and memory values

- *Bits* are represented symbolically (à la bytes in CompCert):

$$b ::= 0 \mid 1 \mid (\mathsf{ptr}\ p)_i \mid \mathsf{indet}$$

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- Memory values have (unique) types

Example

Consider:

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struct T {
  union U { signed char x[2]; int y; } u;
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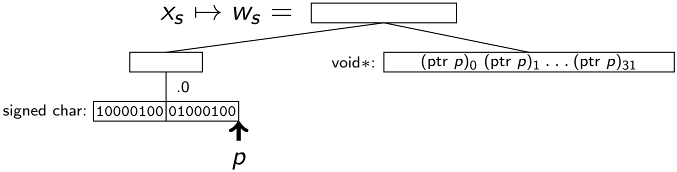

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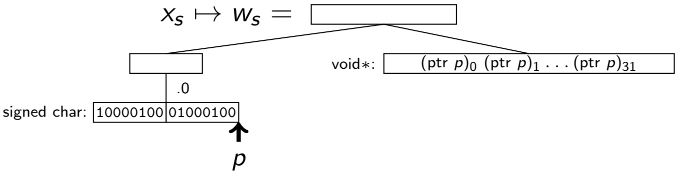

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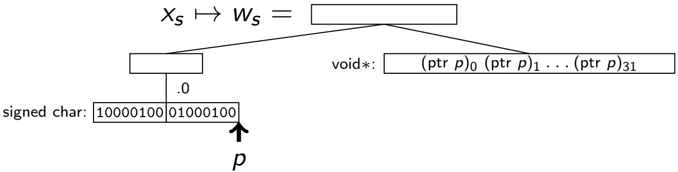

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Hard part: dealing with this choice in *abstract values* and the various operations

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Formalized by decorating pointers with annotations

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Corollary Compilers can perform type based alias analysis

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Theorem "copy by assignment" $\sqsubseteq$ "byte-wise copy"

Formalization in Coq

Type theory is ideal for the combination programming/proving

- *The devil is in the details*, Coq is extremely useful for debugging of definitions
- Useful to prove meta-theoretical properties
- Use of type classes for parametrization by machine integers
- Use of type classes for overloading of notations
- 8.500 lines of code

Future research

- Integration into our operational semantics [K, POPL'14]
  ... and make it (reasonably efficiently) executable
- Memory injections à la CompCert
- Integration into our axiomatic semantics [K, POPL'14]
- Floating point numbers, bit fields, variable length arrays
- The `const`, `volatile` and `restrict` qualifier
- Verification Condition generator in Coq

Questions

Sources: see `http://robbertkrebbers.nl/research/ch2o/`